
MATH 448/548 - Numerical Analysis

Final Project (for undergrad. students)

Additional material

Exact solution for 1D diffusion equation:

$$v(x, t) = \sum_{k=1}^{\infty} \frac{4}{(k\pi)^2} \sin\left(\frac{k\pi}{2}\right) \sin(k\pi x) e^{-D(k\pi)^2 t}, \quad (1)$$

where t is the time, x is the space coordinate and D is the diffusion coefficient.

Matlab code for computing exact solution is given below. In this code the summation has 13 terms (instead of ∞), the time variable is T (so the final time is 1 in this code), $xcoord$ is the space coordinate, J is number of space intervals.

```
T=1;
xcoord=linspace(0,1,J+1)
vxt=zeros(J+1,1);
vxt_sum=zeros(1,J+1);
for k=1:13
    vxt=4*(1/((k*pi).^2))*sin(k*pi/2)*sin(k*pi*xcoord)*exp(-D*(k*pi).^2*T);
    vxt_sum=vxt_sum+vxt;
    sol=vxt_sum
end
```