
MATH 448/548 - Numerical Analysis

Midterm Project

Date: October 19, 2009

Due date: **November 12, 2009**

First Name:

Last Name:

1. [20 points] Derive the Newton-Cotes formula for

$$\int_{1/2}^1 f(x) dx$$

based on fundamental polynomials at nodes $1/2$, $3/4$, and 1 .

Apply this result to evaluate function $f(x) = \cos \pi x$. Explain the results.

2. [20 points] Find the degree of polynomials for which the following quadrature rule is exact:

$$\int_{-1}^1 f(x)dx \approx \frac{1}{9} \left(5f(-\sqrt{3/5}) + 8f(0) + 5f(\sqrt{3/5}) \right)$$

- What is the name of this quadrature rule?
- Evaluate $\int_{1/2}^1 \cos \pi x dx$ using the above rule, is the result different from the one in (1)? Why?

3. [30 points] Consider the following system of linear equations

$$\begin{array}{rclcrcl} 6.25x_1 & -x_2 & +0.5x_3 & = & 7.5, \\ -x_1 & +5x_2 & +2.12x_3 & = & -8.68, \\ 0.5x_1 & +2.12x_2 & +3.6x_3 & = & -0.2. \end{array}$$

- Construct Cholesky factors R and R^T
- Solve the resulting system $RR^T x = f$
- Show that the matrix A of this system is positive definite.

4. [30 points] Consider the following system of linear equations

$$\begin{aligned}1.03x_1 + 0.991x_2 &= 2.51, \\0.991x_1 + 0.943x_2 &= 2.41.\end{aligned}$$

- Solve the system using LU decomposition.
- Compute condition number of the matrix A of this system.
- Solve modified system with right hand side $\vec{b}^* = [2.505, 2.415]^T$.
- Compute relative error of the modified right hand side: $\|\vec{b} - \vec{b}^*\|/\|\vec{b}\|$ and the modified solution $\|\vec{x} - \vec{x}^*\|/\|\vec{x}\|$.
Explain results.