MATH 448/548 "Numerical analysis".

Homework assignment 1.
due date: February 7.

1. Consider the following variation of Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_0)}.$$

   Find C and s s.t.

$$e_{n+1} = Ce_n^s.$$

2. Prove that Newton's iteration will diverge for these functions, regadrless of the choice of real starting point:

   - $f(x) = x^2 + 5$
   - $f(x) = 6x^4 + 3x^2 + \pi$

3. If the secant method is applied to the function $f(x) = x^2 - 2$, with $x_0 = 0$ and $x_1 = 1$ what is x_2?

4. Determine the formula

$$n \geq \frac{\log(b_0 - a_0) - \log \varepsilon}{\log 2} - 1$$

   involving $b_0 - a_0$ and ε for the number of steps that must be taken in the bisection method to guarantee that $|r - c_n| \leq \varepsilon$.

5. Consider equation $4(1 - x^2) - e^x = 0$.

   - Localize positive root of the above equation.
   - Find this root numerically using Newton's method. Use the following stopppig criterion $|x^{n+1} - x^n| \leq 10^{-7}$. Present results as a table showing for each iteration n the approximation of the root x^n and the difference $|x^{n+1} - x^n|$.

   Extra problem for graduate students only.

6. Using Newton's method define the iterative process for calculating $\sqrt[p]{a}$, where $a > 0$ and p is a natural number.