MATH 546
Numerical Analysis of Elliptic PDEs

Homework assignment 5
Date assigned: November 29, 2008
Due date: **December 12, 2008**

- Include a cover page and *this* problem sheet
- Include the printout of your program(s) (if any) for completeness

PROBLEMS:
Consider the one-dimensional Dirichlet problem

$$v_{xx} = f(x) \text{ on } (0,1)$$
$$v(0) = v(1) = 0$$

with $f(x) = cos(x)$ and true solution $v(x) = -f(x)+x*(f(1.0)-1.0)+1.0$.

Implement V-cycle multigrid algorithm to solve the above problem. On each level of multigrid use Gauss-Seidel algorithm to solve the system of equations.

Use the following parameters:

- $m = 7$ – the number of grids to be employed;
- n – number of interior grid points ($n = 2^m - 1$), i.e. finest grid has $n = 127$;
- $k = 2..10$ – the number of iterations to be performed in each Gauss-Seidel method;

Use linear interpolation for prolongation operation I_{2h}^{h} and injection for restriction operation I_{h}^{2h}.

1. Perform multigrid V-cycle for each value of k and print out maximum error ($\max v_i$) for each case.

2. Calculate total number of iterations of Gauss-Seidel method for each multigird algorithm run.
3. Solve the problem using Gauss-Seidel method without multigrid, how many iteartions is required to acheive the same accuracy as V-cycle multigrid?

Short review:

- v denote the exact solution to $Av = f$
- u denote an approximate solution to $Au = f$
- $e = v - u$ is the error incurred in using u to approx v
- r is the residual associated with u; $r = f - Au$. It follows that $Ae = r$
- If we approximately solve for e, then an improved estimate for v is obtained from $u + e$.

Coarse Grid Correction:

- Iterate on G_h to obtain u_h as an approximation to $Av = f$
- Compute the residual $r = f - Au_h$
- Iterate on G_{2h} to obtain an approximate solution to $Ae = r$
- Correct the approximation obtained on G_h by replacing v_h with $u_h + e$

A *recursive* application of Coarse Grid Correction is the basis for the Multigrid V-Cycle.