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# MATH 546

## Numerical Analysis of Elliptic PDEs

### Homework assignment 3

*Date assigned:* November 3, 2008

*Due date:* **November 14, 2008**

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- Include a cover page and *this* problem sheet
- Include the printout of your program(s) (if any) for completeness

#### PROBLEMS:

Consider the following Neumann problem:

$$\begin{cases} -\Delta u = f \equiv e^{x+y} & \text{in } \Omega, \\ \frac{\partial u}{\partial n} = g \equiv \frac{1}{2}e^{x+y} & \text{on } \partial\Omega. \end{cases} \quad (1)$$

Let  $\Omega$  be square  $(0, 1) \times (0, 1)$ .

1. Consider first order approximation of boundary conditions
  - Construct finite-difference approximation of problem (1) with  $\Delta x = \Delta y = 1/4$
  - Build matrix  $A$  corresponding to the approximate problem.
2. Consider second order approximation of boundary conditions
  - Construct finite-difference approximation of problem (1) with  $\Delta x = \Delta y = 1/4$
  - Build matrix  $A$  corresponding to the approximate problem.
3. Verify that (1) satisfies the compatibility condition.
4. Calculate eigenvalues and eigenvectors associated with Jacobi iteration matrix for matrix  $A$  computed in parts 1 and 2. Based on your result what can you say about convergence of Jacobi iteration applied to (1)?