MATH 546  
Numerical Analysis of Elliptic PDEs  

Homework assignment 2  
Date assigned: October 3, 2008  
Due date: October 17, 2008  

• Include a cover page and *this* problem sheet  
• Include the printout of your program(s) (if any) for completeness  

PROBLEM:  
Consider the following Dirichlet problem:  
\[
\begin{align*}
-\Delta u &= -\sin \pi x \sin 2\pi y \text{ in } \Omega \\
u &= 0 \text{ on } \partial \Omega.
\end{align*}
\]  
(1)  
Let \( \Omega \) be square \((0, 1) \times (0, 1)\).  
• Use Gauss-Seidel and SOR schemes to find an approximate solution of problem (1) with \( \Delta x = \Delta y = 1/10 \).  
• Use two stopping criteria: difference of two successive iterations and residual in \( l_2 \) and sup-norm with tolerance \( 10^{-5} \).  
• What is the optimal value of \( \omega_b \) for the SOR method?  
• For each scheme show the following information  

| Iteration number | \( ||x^{k+1} - x^k||_2 \) | \( ||r^{k+1}||_2 \) | \( ||x^{k+1} - x^k||_\infty \) | \( ||r^{k+1}||_\infty \) |
|------------------|-----------------|-----------------|-----------------|-----------------|

• Plot the solutions you obtained