MATH 546
Numerical Analysis of Evolution PDEs

Homework assignment 1
Date assigned: September 12, 2008
Due date: **September 26, 2008**

- Include a cover page and *this* problem sheet
- Include the printout of your program(s) (if any) for completeness

PROBLEMS:

1. Consider the following Neumann problem:

$$\begin{cases} -\Delta u = f \text{ in } \Omega, \\ \frac{\partial u}{\partial \vec{n}} = h \text{ on } \partial\Omega. \end{cases} \tag{1}$$

   Prove the integrability condition:

$$\int\int_{\Omega} f dx = -\int_{\partial\Omega} h ds.$$

2. Using Taylor series expansion find the order of the following formulas:
   - $u'(x) \approx \frac{1}{12\Delta x}\left(-u(x+\Delta x) + 8u(x+\Delta x) - 8u(x-\Delta x) + u(x-2\Delta x)\right)$
   - $u''(x) \approx \frac{1}{\Delta x^2}\left(u(x) - 2u(x+\Delta x) + u(x+2\Delta x)\right)$
   - $u'(x) \approx \frac{1}{2\Delta x}\left(u(x+\Delta x) - u(x-\Delta x)\right)$

3. Apply method of undetermined coefficients to derive second order scheme for $\frac{\partial u}{\partial x}$ using three points in the following way:

$$u_{xx} \approx c_1 u(k) + c_2 u(k+1) + c_3 u(k+2).$$

   Can you derive second order scheme using two points?

4. Consider the following Dirichlet problem:

$$\begin{cases} -\Delta u = f \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega. \end{cases} \tag{2}$$

   Let Ω be square $(0,1) \times (0,1)$.
   - Construct finite-difference approximation of problem (2) with $\Delta x = \Delta y = 1/5$.
   - Build matrix A corresponding to the approximate problem.
   - Show that A is positive-definite.