

To facilitate the description of this procedure, some economy of notation is desirable. The same notation is used for the computer implementation of the resulting algorithm. We call the right-side vector of the residual equation f^{2h} , rather than r^{2h} , because it is just another right-side vector. Instead of calling the solution of the residual equation e^{2h} , we use u^{2h} because it is just a solution vector. We can then use v^{2h} to denote approximations to u^{2h} . These changes simplify the notation, but it is still important to remember the meaning of these variables.

One more point needs to be addressed: what initial guess do we use for v^{2h} on the first visit to Ω^{2h} ? Because there is presumably no information available about the solution, u^{2h} , we simply choose $v^{2h} = 0$. Here then is the two-grid correction scheme, now imbedded within itself. We assume that there are $l > 1$ grids with grid spacings $h, 2h, 4h, \dots, Lh = 2^{l-1}h$.

V-Cycle Scheme

- $v^h \leftarrow V^h(v^h, f^h)$
- Relax on $A^h u^h = f^h$ ν_1 times with initial guess v^h .
- Compute $f^{2h} = I_h^{2h} r^h$.
- Relax on $A^{2h} u^{2h} = f^{2h}$ ν_1 times with initial guess $v^{2h} = 0$.
- Compute $f^{4h} = I_{2h}^{4h} r^{2h}$.
- Relax on $A^{4h} u^{4h} = f^{4h}$ ν_1 times with initial guess $v^{4h} = 0$.
- Compute $f^{8h} = I_{4h}^{8h} r^{4h}$.
- Solve $A^{Lh} u^{Lh} = f^{Lh}$.

- Correct $v^{4h} \leftarrow v^{4h} + I_{8h}^{4h} v^{8h}$.
- Relax on $A^{4h} u^{4h} = f^{4h}$ ν_2 times with initial guess v^{4h} .
- Correct $v^{2h} \leftarrow v^{2h} + I_{4h}^{2h} v^{4h}$.
- Relax on $A^{2h} u^{2h} = f^{2h}$ ν_2 times with initial guess v^{2h} .
- Correct $v^h \leftarrow v^h + I_h^h v^{2h}$.
- Relax on $A^h u^h = f^h$ ν_2 times with initial guess v^h .

The algorithm telescopes down to the coarsest grid, which can consist of one or a few interior grid points, then works its way back to the finest grid. Figure 3.6(a) shows the schedule for the grids in the order in which they are visited. Because of the pattern in this diagram, this algorithm is called the V-cycle. It is our first true multigrid method.

Not surprisingly, the V-cycle has a compact recursive definition, which is given as follows.

V-Cycle Scheme (Recursive Definition)

- $v^h \leftarrow V^h(v^h, f^h)$.
- 1. Relax ν_1 times on $A^h u^h = f^h$ with a given initial guess v^h .

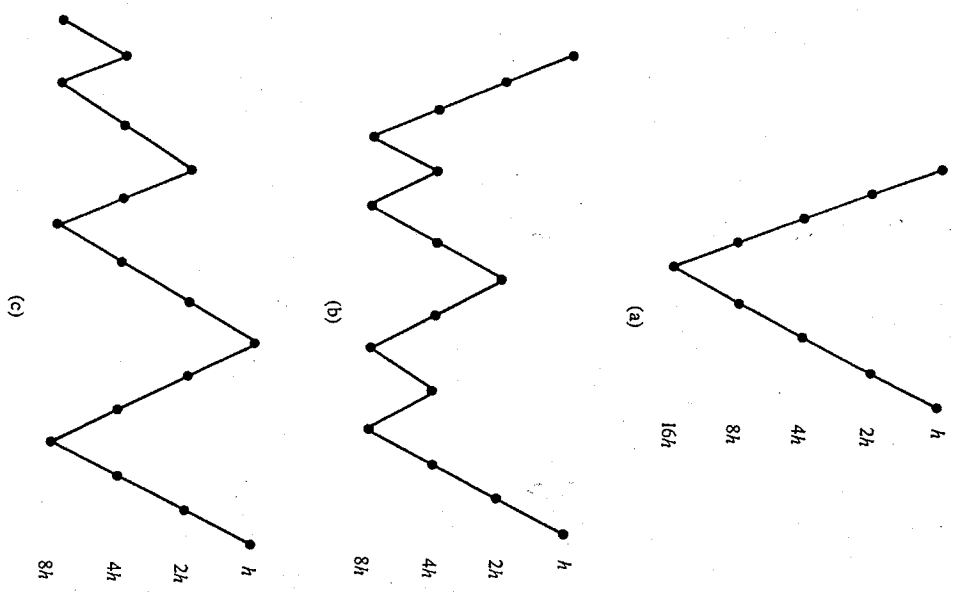


Figure 3.6. Schedule of grids for (a) V-cycle, (b) W-cycle, and (c) FMG scheme, all on four levels.

- 2. If $\Omega^h =$ coarsest grid, then go to step 4.
- Else

$$f^{2h} \leftarrow I_h^{2h} (f^h - A^h v^h),$$

$$v^{2h} \leftarrow 0,$$

$$v^{2h} \leftarrow V^{2h}(v^{2h}, f^{2h}).$$

- 3. Correct $v^h \leftarrow v^h + I_h^h v^{2h}$.
- 4. Relax ν_2 times on $A^h u^h = f^h$ with initial guess v^h .