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# MATH 441/541 - Applied Mathematics II

## Final Exam

*Date assigned:* April 29, 2009

**Due date:** May 7, 2009

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**First Name:**

**Last Name:**

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1. Find if the following function harmonic

$$u = xy$$

if yes find a corresponding analytic function.

2. Evaluate the integral:

$$\int_C \operatorname{Re} z \, dz, \quad C \text{ is parabola } y = x^2 \text{ from } 0 \text{ to } 1 + i$$

3. Integrate counterclockwise using Cauchy integral formula

$$\int_C \frac{e^{3z}}{3z - i} dz, \quad C : |z| = 1.$$

4. Check if the following sequence is bounded, convergent

$$z_n = e^{-n\pi i/4}$$

5. Find the inverse  $z = z(w)$ . Check the result by solving  $z(w)$  from  $w$ .

$$w = \frac{4z + i}{-3iz + 1}$$

6. Find the linear fractional transformation that maps the given three points onto the three given points in respective order.

$$0, 1, 2 \text{ onto } 1, 1/2, 1/3$$

7. Find and sketch the potential. Find complex potential. Between parallel plates at  $x = -3$  and  $3$ , potentials  $140$  V and  $260$  V, respectively.
8. Find the temperature between two parallel plates  $y = 0$  and  $y = d$  kept at temperatures  $0^\circ\text{C}$  and  $100^\circ\text{C}$ , respectively.
9. Show the Choletsky's factorization and solve:

$$\begin{aligned} 9x_1 + 6x_2 + 12x_3 &= 87, \\ 6x_1 + 13x_2 + 11x_3 &= 118, \\ 12x_1 + 11x_2 + 26x_3 &= 154. \end{aligned}$$

10. Maximize the given objective function  $f$  subject to constraints:

$$f = 2x_1 + 3x_2, \quad 4x_1 + 3x_2 \geq 12, \quad x_1 - x_2 \geq -3, \quad x_2 \leq 6, \quad 2x_1 - 3x_2 \leq 0.$$