This form (6) of (1) suggests **Picard's iteration method**[8], which is defined by

$$(7) \quad y_n(x) = y_0 + \int_{x_0}^{x} f(t, y_{n-1}(t))\, dt, \quad n = 1, 2, \cdots.$$

It gives approximations $y_1, y_2, y_3, \cdots$ of the unknown solution $y$ of (1). Indeed, you obtain $y_1$ by substituting $y = y_0$ on the right and integrating—this is the first step—, then $y_2$ by substituting $y = y_1$ on the right and integrating—this is the second step—, and so on. Write a program of the iteration that gives a printout of the first approximations $y_0, y_1, \cdots, y_N$ as well as their graphs on common axes. Try your program on two initial value problems of your own choice.

**(B)** Apply the iteration to $y' = x + y$, $y(0) = 0$. Also solve the problem exactly.

**(C)** Apply the iteration to $y' = 2y^2$, $y(0) = 1$. Also solve the problem exactly.

**(D)** Find all solutions of $y' = 2\sqrt{y}$, $y(1) = 0$. Which of them does Picard's iteration approximate?

**(E)** Experiment with the conjecture that Picard's iteration converges to the solution of the problem for any initial choice of $y$ in the integrand in (7) (leaving $y_0$ outside the integral as it is). Begin with a simple ODE and see what happens. When you are reasonably sure, take a slightly more complicated ODE and give it a try.

## CHAPTER 1 REVIEW QUESTIONS AND PROBLEMS

1. Explain the terms *ordinary differential equation (ODE), partial differential equation (PDE), order, general solution,* and *particular solution.* Give examples. Why are these concepts of importance?
2. What is an initial condition? How is this condition used in an initial value problem?
3. What is a homogeneous linear ODE? A nonhomogeneous linear ODE? Why are these equations simpler than nonlinear ODEs?
4. What do you know about direction fields and their practical importance?
5. Give examples of mechanical problems that lead to ODEs.
6. Why do electric circuits lead to ODEs?
7. Make a list of the solution methods considered. Explain each method with a few short sentences and illustrate it by a typical example.
8. Can certain ODEs be solved by more than one method? Give three examples.
9. What are integrating factors? Explain the idea. Give examples.
10. Does every first-order ODE have a solution? A general solution? What do you know about uniqueness of solutions?

### 11–14 DIRECTION FIELDS

Graph a direction field (by a CAS or by hand) and sketch some of the solution curves. Solve the ODE exactly and compare.

**11.** $y' = 1 + 4y^2$ **12.** $y' = 3y - 2x$

**13.** $y' = 4y - y^2$ **14.** $y' = 16x/y$

### 15–26 GENERAL SOLUTION

Find the general solution. Indicate which method in this chapter you are using. Show the details of your work.

**15.** $y' = x^2(1 + y^2)$

**16.** $y' = x(y - x^2 + 1)$

**17.** $yy' + xy^2 = x$

**18.** $-\pi \sin \pi x \cosh 3y\, dx + 3 \cos \pi x \sinh 3y\, dy = 0$

**19.** $y' + y \sin x = \sin x$ **20.** $y' - y = 1/y$

**21.** $3 \sin 2y\, dx + 2x \cos 2y\, dy = 0$

**22.** $xy' = x \tan (y/x) + y$

**23.** $(y \cos xy - 2x)\, dx + (x \cos xy + 2y)\, dy = 0$

**24.** $xy' = (y - 2x)^2 + y$ (Set $y - 2x = z$.)

**25.** $\sin (y - x)\, dx + [\cos (y - x) - \sin (y - x)]\, dy = 0$

**26.** $xy' = (y/x)^3 + y$

### 27–32 INITIAL VALUE PROBLEMS

Solve the following initial value problems. Indicate the method used. Show the details of your work.

**27.** $yy' + x = 0, \quad y(3) = 4$

**28.** $y' - 3y = -12y^2, \quad y(0) = 2$

**29.** $y' = 1 + y^2, \quad y(\frac{1}{4}\pi) = 0$

**30.** $y' + \pi y = 2b \cos \pi x, \quad y(0) = 0$

**31.** $(2xy^2 - \sin x)\, dx + (2 + 2x^2 y)\, dy = 0, \; y(0) = 1$

**32.** $[2y + y^2/x + e^x(1 + 1/x)]\, dx + (x + 2y)\, dy = 0,$ $y(1) = 1$

[8]EMILE PICARD (1856–1941), French mathematician, also known for his important contributions to complex analysis (see Sec. 16.2 for his famous theorem). Picard used his method to prove Theorems 1 and 2 as well as the convergence of the sequence (7) to the solution of (1). In precomputer times the iteration was of little *practical* value because of the integrations.