EXAMPLE 5 Stable and Unstable Equilibrium Solutions. "Phase Line Plot"

The ODE $y' = (y - 1)(y - 2)$ has the stable equilibrium solution $y_1 = 1$ and the unstable $y_2 = 2$, as the direction field in Fig. 19 suggests. The values y_1 and y_2 are the zeros of the parabola $f(y) = (y - 1)(y - 2)$ in the figure. Now, since the ODE is autonomous, we can "condense" the direction field to a "phase line plot" giving y_1 and y_2, and the direction (upward or downward) of the arrows in the field, and thus giving information about the stability or instability of the equilibrium solutions. ■

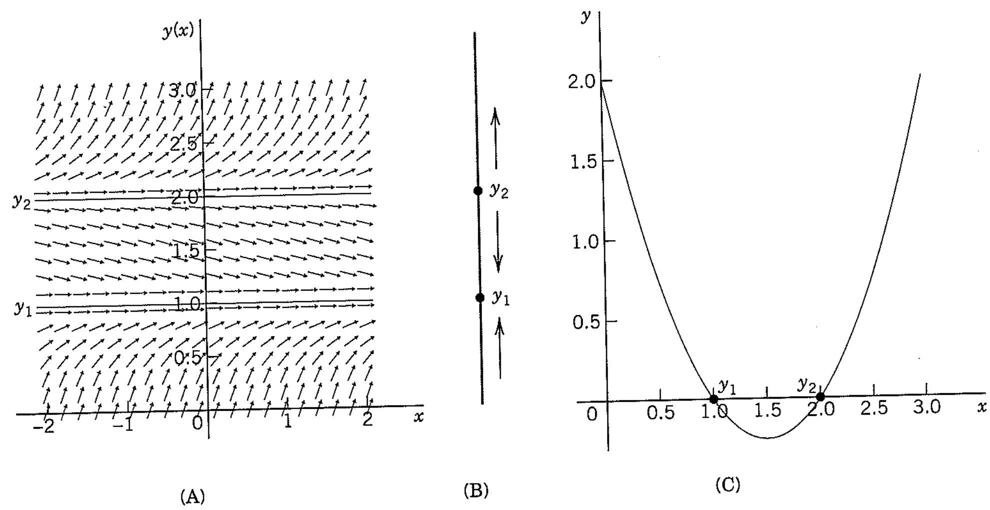

Fig. 19. Example 5. (A) Direction field. (B) "Phase line". (C) Parabola $f(y)$

A few further population models will be discussed in the problem set. For some more details of population dynamics, see C. W. Clark, *Mathematical Bioeconomics,* New York, Wiley, 1976.

Further important applications of linear ODEs follow in the next section.

PROBLEM SET 1.5

1. (CAUTION!) Show that $e^{-\ln x} = 1/x$ (not $-x$) and $e^{-\ln(\sec x)} = \cos x$.
2. **(Integration constant)** Give a reason why in (4) you may choose the constant of integration in $\int p\,dx$ to be zero.

3–17 GENERAL SOLUTION. INITIAL VALUE PROBLEMS

Find the general solution. If an initial condition is given, find also the corresponding particular solution and graph or sketch it. (Show the details of your work.)

3. $y' + 3.5y = 2.8$
4. $y' = 4y + x$
5. $y' + 1.25y = 5, \quad y(0) = 6.6$
6. $x^2y' + 3xy = 1/x, \quad y(1) = -1$
7. $y' + ky = e^{2kx}$
8. $y' + 2y = 4\cos 2x, \quad y(\frac{1}{4}\pi) = 2$
9. $y' = 6(y - 2.5)\tanh 1.5x$
10. $y' + 4x^2y = (4x^2 - x)e^{-x^2/2}$
11. $y' + 2y\sin 2x = 2e^{\cos 2x}, \quad y(0) = 0$
12. $y'\tan x = 2y - 8, \quad y(\frac{1}{2}\pi) = 0$
13. $y' + 4y\cot 2x = 6\cos 2x, \quad y(\frac{1}{4}\pi) = 2$
14. $y' + y\tan x = e^{-0.01x}\cos x, \quad y(0) = 0$
15. $y' + y/x^2 = 2xe^{1/x}, \quad y(1) = 13.86$
16. $y'\cos^2 x + 3y = 1, \quad y(\frac{1}{4}\pi) = \frac{4}{3}$
17. $x^3y' + 3x^2y = 5\sinh 10x$