Differentiate this with respect to y and use (4b) to get

$$\frac{\partial u}{\partial y} = x + \frac{dk}{dy} = N = x - e^{-y}, \qquad \frac{dk}{dy} = -e^{-y}, \qquad k = e^{-y} + c^*.$$

Hence the general solution is

$$u(x, y) = e^x + xy + e^{-y} = c.$$

Step 3. Particular solution. The initial condition $y(0) = 1$ gives $u(0, -1) = 1 + 0 + e = 3.72$. Hence the answer is $e^x + xy + e^{-y} = 1 + e = 3.72$. Figure 15 shows several particular solutions obtained as level curves of $u(x, y) = c$, obtained by a CAS, a convenient way in cases in which it is impossible or difficult to cast a solution into explicit form. Note the curve that (nearly) satisfies the initial condition.

Step 4. Checking. Check by substitution that the answer satisfies the given equation as well as the initial condition. ■

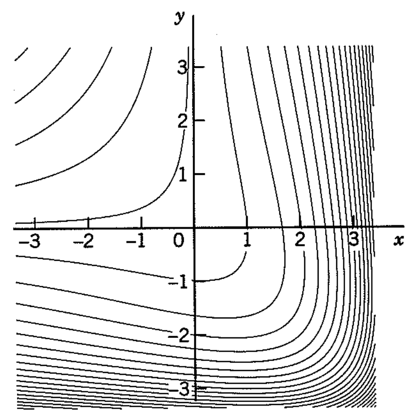

Fig. 15. Particular solutions in Example 5

PROBLEM SET 1.4

1–20 EXACT ODEs. INTEGRATING FACTORS

Test for exactness. If exact, solve. If not, use an integrating factor as given or find it by inspection or from the theorems in the text. Also, if an initial condition is given, determine the corresponding particular solution.

1. $x^3\,dx + y^3\,dy = 0$ **2.** $(x - y)(dx - dy) = 0$

3. $-\pi \sin \pi x \sinh y\,dx + \cos \pi x \cosh y\,dy = 0$

4. $(e^y - ye^x)\,dx + (xe^y - e^x)\,dy = 0$

5. $9x\,dx + 4y\,dy = 0$

6. $e^x(\cos y\,dx - \sin y\,dy) = 0$

7. $e^{-2\theta}\,dr - 2re^{-2\theta}\,d\theta = 0$

8. $(2x + 1/y - y/x^2)\,dx + (2y + 1/x - x/y^2)\,dy = 0$

9. $(-y/x^2 + 2\cos 2x)\,dx + (1/x - 2\sin 2y)\,dy = 0$

10. $-2xy \sin(x^2)\,dx + \cos(x^2)\,dy = 0$

11. $-y\,dx + x\,dy = 0$

12. $(e^{x+y} - y)\,dx + (xe^{x+y} + 1)\,dy = 0$

13. $-3y\,dx + 2x\,dy = 0, \quad F(x, y) = y/x^4$

14. $(x^4 + y^2)\,dx - xy\,dy = 0, \quad y(2) = 1$

15. $e^{2x}(2\cos y\,dx - \sin y\,dy) = 0, \quad y(0) = 0$

16. $-\sin xy\,(y\,dx + x\,dy) = 0, \quad y(1) = \pi$

17. $(\cos \omega x + \omega \sin \omega x)\,dx + e^x\,dy = 0, y(0) = 1$

18. $(\cos xy + x/y)\,dx + (1 + (x/y)\cos xy)\,dy = 0$

19. $e^{-y}\,dx + e^{-x}(-e^{-y} + 1)\,dy = 0, \quad F = e^{x+y}$

20. $(\sin y \cos y + x\cos^2 y)\,dx + x\,dy = 0$

21. Under what conditions for the constants A, B, C, D is $(Ax + By)\,dx + (Cx + Dy)\,dy = 0$ exact? Solve the exact equation.