PROBLEM SET 1.3

1. **(Constant of integration)** An arbitrary constant of integration must be introduced immediately when the integration is performed. Why is this important? Give an example of your own.

2–9 GENERAL SOLUTION

Find a general solution. Show the steps of derivation. Check your answer by substitution.

2. $y' + (x + 2)y^2 = 0$
3. $y' = 2 \sec 2y$
4. $y' = (y + 9x)^2 \quad (y + 9x = v)$
5. $yy' + 36x = 0$
6. $y' = (4x^2 + y^2)/(xy)$
7. $y' \sin \pi x = y \cos \pi x$
8. $xy' = \frac{1}{2}y^2 + y$
9. $y' e^{\pi x} = y^2 + 1$

10–19 INITIAL VALUE PROBLEMS

Find the particular solution. Show the steps of derivation, beginning with the general solution. (L, R, b are constants.)

10. $yy' + 4x = 0,\ y(0) = 3$
11. $dr/dt = -2tr,\ r(0) = r_0$
12. $2xyy' = 3y^2 + x^2,\ y(1) = 2$
13. $L\ dI/dt + RI = 0,\ I(0) = I_0$
14. $y' = y/x + (2x^3/y)\cos(x^2),\ y(\sqrt{\pi/2}) = \sqrt{\pi}$
15. $e^{2x}y' = 2(x + 2)y^3,\ y(0) = 1/\sqrt{5} \approx 0.45$
16. $xy' = y + 4x^5 \cos^2(y/x),\ y(2) = 0$
17. $y' x \ln x = y,\ y(3) = \ln 81$
18. $dr/d\theta = b[(dr/d\theta)\cos\theta + r \sin\theta],\ r(\frac{1}{2}\pi) = \pi,\ 0 < b < 1$
19. $yy' = (x - 1)e^{-y^2},\ y(0) = 1$

20. **(Particular solution)** Introduce limits of integration in (3) such that y obtained from (3) satisfies the initial condition $y(x_0) = y_0$. Try the formula out on Prob. 19.

21–36 APPLICATIONS, MODELING

21. **(Curves)** Find all curves in the xy-plane whose tangents all pass through a given point (a, b).
22. **(Curves)** Show that any (nonvertical) straight line through the origin of the xy-plane intersects all solution curves of $y' = g(y/x)$ at the same angle.
23. **(Exponential growth)** If the growth rate of the amount of yeast at any time t is proportional to the amount present at that time and doubles in 1 week, how much yeast can be expected after 2 weeks? After 4 weeks?
24. **(Population model)** If in a population of bacteria the birth rate and death rate are proportional to the number of individuals present, what is the population as a function of time? Figure out the limiting situation for increasing time and interpret it.
25. **(Radiocarbon dating)** If a fossilized tree is claimed to be 4000 years old, what should be its ${}_6C^{14}$ content expressed as a percent of the ratio of ${}_6C^{14}$ to ${}_6C^{12}$ in a living organism?
26. **(Gompertz growth in tumors)** The Gompertz model is $y' = -Ay \ln y$ $(A > 0)$, where $y(t)$ is the mass of tumor cells at time t. The model agrees well with clinical observations. The declining growth rate with increasing $y > 1$ corresponds to the fact that cells in the interior of a tumor may die because of insufficient oxygen and nutrients. Use the ODE to discuss the growth and decline of solutions (tumors) and to find constant solutions. Then solve the ODE.
27. **(Dryer)** If wet laundry loses half of its moisture during the first 5 minutes of drying in a dryer and if the rate of loss of moisture is proportional to the moisture content, when will the laundry be practically dry, say, when will it have lost 95% of its moisture? First guess.
28. **(Alibi?)** Jack, arrested when leaving a bar, claims that he has been inside for at least half an hour (which would provide him with an alibi). The police check the water temperature of his car (parked near the entrance of the bar) at the instant of arrest and again 30 minutes later, obtaining the values 190°F and 110°F, respectively. Do these results give Jack an alibi? (Solve by inspection.)
29. **(Law of cooling)** A thermometer, reading 10°C, is brought into a room whose temperature is 23°C. Two minutes later the thermometer reading is 18°C. How long will it take until the reading is practically 23°C, say, 22.8°C? First guess.
30. **(Torricelli's law)** How does the answer in Example 5 (the time when the tank is empty) change if the diameter of the hole is doubled? First guess.
31. **(Torricelli's law)** Show that (7) looks reasonable inasmuch as $\sqrt{2gh(t)}$ is the speed a body gains if it falls a distance h (and air resistance is neglected).
32. **(Rope)** To tie a boat in a harbor, how many times must a rope be wound around a bollard (a vertical rough cylindrical post fixed on the ground) so that a man holding one end of the rope can resist a force exerted by the boat one thousand times greater than the man can exert? First guess. Experiments show that the change ΔS of the force S in a small portion of the rope is proportional to S and to the small angle $\Delta\phi$ in Fig. 13. Take the proportionality constant 0.15.