In Exercise 11-12, determine the value(s) of $h$ such that the matrix

$$
\begin{bmatrix}
0 & 2 & 0 \\
0 & 1 & 0 \\
0 & -1 & 0 \\
\end{bmatrix}
$$

is the augmented matrix of a consistent linear system.

**Exercise 11.** Do the three planes $x + 2z = 1$, $x + 2y = 1$, and $x + 3y = 1$ have a common point of intersection? If so, find the corner.

**Exercise 12.** Do the three lines $x - 1z = 1$, $x - 2z = 4$, and $x - 3z = 4$ have a common point of intersection? If so, find the corner.

Do not completely solve the systems in Exercises 15 and 16. Determine if the systems in Exercises 15 and 16 are consistent.

15. Consider each matrix in Exercises 5 and 6 to determine the system of linear equations that should be solved in the process of solving the system. State in words the next two elementary row operations that must be performed. Then write the augmented matrix for the new system of linear equations.

16. Find the point of intersection of the lines $x - 2y = 1$ and $z = 0$.

Solve the systems in Exercises 11-14.

10. Find the point of intersection of the lines $x + 2y = 1$ and $z = 0$.

**Systems of Linear Equations**

1.1 Exercises

[Graph of linear equations and matrices]
In Exercises 23 and 24, key statements from this section are either quoted directly, restated slightly (but still true), or altered in some way that makes them false in some cases. Mark each statement True or False, and justify your answer. (If true, give the approximate location where a similar statement appears, or refer to a definition or theorem. If false, give the location of a statement that has been quoted or used incorrectly, or cite an example that shows the statement is not true in all cases.) Similar true/false questions will appear in many sections of the text.

23. a. Every elementary row operation is reversible. \( \checkmark \)
   b. A 5 x 6 matrix has six rows. \( \times \)
   c. The solution set of a linear system involving variables \( x_1, \ldots, x_6 \) is a list of numbers \( (s_1, \ldots, s_6) \) that makes each equation in the system a true statement when the values \( s_1, \ldots, s_6 \) are substituted for \( x_1, \ldots, x_6 \), respectively. \( \checkmark \)
   d. Two fundamental questions about a linear system involve existence and uniqueness. \( \times \)

24. a. Elementary row operations on an augmented matrix never change the solution set of the associated linear system. \( \checkmark \)
   b. Two matrices are row equivalent if they have the same number of rows. \( \checkmark \)
   c. An inconsistent system has more than one solution. \( \times \)
   d. Two linear systems are equivalent if they have the same solution set. \( \checkmark \)

25. Find an equation involving \( g, h, \) and \( k \) that makes this augmented matrix correspond to a consistent system:

\[
\begin{bmatrix}
1 & -4 & 7 & g \\
0 & 3 & -5 & h \\
-2 & 5 & -9 & k
\end{bmatrix}
\]

26. Construct three different augmented matrices for linear systems whose solution set is \( x_1 = -2, x_2 = 1, x_3 = 0 \).

27. Suppose the system below is consistent for all possible values of \( f \) and \( g \). What can you say about the coefficients \( c \) and \( d \)? Justify your answer.

\[
x_1 + 3x_2 = f \\
cx_1 + dx_2 = g
\]

28. Suppose \( a, b, c, \) and \( d \) are constants such that \( a \) is not zero and the system below is consistent for all possible values of \( f \) and \( g \). What can you say about the numbers \( a, b, c, \) and \( d \)? Justify your answer.

\[
ax_1 + bx_2 = f \\
cx_1 + dx_2 = g
\]

In Exercises 29–32, find the elementary row operation that transforms the first matrix into the second, and then find the reverse row operation that transforms the second matrix into the first.

29. \[
\begin{bmatrix}
0 & -2 & 5 \\
1 & 4 & -7 \\
3 & -1 & 6
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 4 & -7 \\
0 & -2 & 5 \\
3 & -1 & 6
\end{bmatrix}
\]

30. \[
\begin{bmatrix}
1 & 3 & -4 \\
0 & -2 & 6 \\
0 & -5 & 9
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 3 & -4 \\
0 & 1 & -3 \\
0 & -5 & 9
\end{bmatrix}
\]

31. \[
\begin{bmatrix}
1 & -2 & 1 \\
0 & 5 & -2 \\
4 & -1 & 3
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & -2 & 1 \\
0 & 5 & -2 \\
4 & -1 & 3
\end{bmatrix}
\]

32. \[
\begin{bmatrix}
1 & 2 & -5 \\
0 & 1 & -3 \\
0 & -3 & 9
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 2 & -5 \\
0 & 1 & -3 \\
0 & 0 & -1
\end{bmatrix}
\]

An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let \( T_1, \ldots, T_4 \) denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes—to the left, above, to the right, and below.\(^3\) For instance,

\[
T_1 = (10 + 20 + T_2 + T_3)/4 \quad \text{or} \quad 4T_1 - T_2 - T_4 = 30
\]

\[\text{Figure 1: Heat Transfer Problem}\]