

Writing Proofs

By Nathan Hamlin

As we approach our first exam, here are some general recommendations for writing proofs, some of which are more immediately important than others. When we first start writing proofs, we are often more familiar with writing for other contexts.

In an idealized mathematical classroom setting, you can expect that everyone is mostly familiar with the definitions and theorems, that they are able to follow a good argument, even if it requires some effort on their part, and that if they are shown a good proof, they will accept the result as true. On these assumptions, a good proof has good reasons and good reasoning, is clear and concise, has a good opening and conclusion, and effectively uses the relevant material to bring the reader to relative certainty regarding the result. You can assume that your reader is *interested in the content*, but *disinterested with respect to the result*.

As you move into the rough and tumble of professional life, there may be occasions where these expectations are unrealistic, *even if they are the conditions you prefer*, and as you speak and write papers you may need to adjust your expectations to social realities. This is especially true if you are doing interdisciplinary work, even within the field of mathematics. You might start with a good proof as described above, and perhaps people would not be convinced, even at a math conference with able mathematicians! Next, you might acquire empirical evidence that supports your proof. Even if this became widely known this might not convince people in your field of the truth of your result! *You might have to approach things differently if this was the case*.

Let me give some examples of what I mean. Perhaps if your result was *known to be true*, many powerful people would feel obligated to make different decisions about pre-existing infrastructure that had many millions of dollars invested in it. Mathematicians are generally people, and sometimes people don't like to say that they are really being motivated by money. Should you point out, in your paper, the financial interests and infrastructure that are likely keeping your result from being accepted by some readers? That might be pretty unpopular with some of the very people you are trying to convince!

Perhaps some of your readers think they know more physics, philosophy, and linguistics than they actually do and they are resistant to your result on the basis of lay misunderstandings of these fields. Perhaps your result touches *very keenly* on application, and is *deep enough*, that it requires an unusual amount of clarity surrounding things like hardware and the use of mathematical symbols and notation. Should you clarify these issues for you reader, or at least point them out? Some people might complain that you are not writing a mathematics paper!

What if the kinds of *choices* that your work would make possible on a public level are considered threatening or even **evil** by powerful religious interests? What if you *came* from such circles, and you *knew* that these people generally resist certain *kinds* of mathematical results for religious reasons, while giving other reasons (lying about their intentions)? Should you point that out in your paper? Sounds

impossible! Given our religious history, you might want to do this *very carefully*. Even so, you might make a lot of people mad.

Perhaps there was a great mathematician who lived a while back, and he was very smart about many things, and speculated about many matters outside his area of expertise. Perhaps he is still well loved, but more importantly, many people who are not attached to that man would feel like they were risking their careers if they were to challenge him publicly, or deal kindly with someone who did. If mathematics is going to be different than certain other areas of human life, perhaps even than certain other fields in the Academy, might it be important for someone at some point to stand up and say “Hilbert was a great man, but he was not right about everything?” If a great mathematician and his followers were men and women of humility, this might not be a problem. But perhaps you can be great without being humble. In my experience, you can be *spit upon*, and even threatened, if you say something like this, in so many words, at an international math conference. So if you are going to include this in a mathematics paper that is going to be *very widely read*, it might be best to approach it gently. But even so, might this make it *very hard* to get your paper accepted by a prestigious journal?

Perhaps your result, though immensely important, and in an important area mathematics, is from an area of mathematics that many mathematicians do not *think* is very important, even though they probably would never *say* that. In additions to your reasons, might you need some *rhetoric* to help people *see* that it is important? A result could be *true*, after all, but not one we need to think very much about, or make use of in our public decision making. But doesn't rhetoric take us far away from mathematics? Another reason your paper might not be accepted by the most prestigious journals.

Now, suppose your result is both true **and** of immense theoretical and practical importance. It seems possible, under the above conditions, that it could “slip through the cracks,” especially *if it was the first thing you did as a professional mathematician*. It might take you a long time to figure out *why* it was not generally accepted (in terms of implementing the results), and *how* to help people see what they needed to see. This wouldn't be mathematical work exactly (another reason people might disapprove) but it might be *very important work nonetheless*, and *a service to the mathematical community*, that *could only really be done by a mathematician*. If you move the stones to dig out your result, this might irritate a lot of people, including in related fields. Sometimes it is important to do things *even if you know some people will be irritated*. And, if the conditions we have discussed above hold, *other people* might be to blame for your work falling through the cracks. If you dig it out, they might look bad. But perhaps a man or woman is allowed to dig out his or her own work, even in the Academy.