

Predicates

By Nathan Hamlin

$$P(x_1, x_2, x_3, \dots, x_n)$$

A predicate is a sentence with *variables*, which when the variables are **fixed** to be certain constants, the sentence becomes a statement and is thus either true or false.

In general, the variables cannot take on *any* value, but there might be more than one allowed value for the variable in a certain context.

A predicate might be true when evaluated with certain constants; false when evaluated with others.

If the variables are not evaluated, but are left open, the sentence is neither true nor false.

Thus, a series of predicates is neither true nor false in the abstract: it is only true or false when particular values are assigned to the variables.

If the universal set is fundamentally altered, the truth set of a predicate or series of predicates might be altered as well.

A predicate that is indeterminate for all values of all variables, or whose truth set is the empty set for such, might become determinate or even true with the addition of one or more constants to either the universal set or the range of one or more of the variables.

Thus, a predicate that is indeterminate, or which has an empty truth set until time t , might *after* time t become determinate and true.

Purely mathematical entities do not change over time, but the *accepted* or acceptable definitions of words used within the mathematical community do.

Sometimes we determine the value of the variables from the context; thus, to use a predicate effectively in a formal proof or demonstration the variables must be clearly defined, and the possible constants enumerated.

In an informal argument, we might leave out the value of the variable, if we think the reader knows the likely value, or if we would prefer it only to be understood by someone who does.

Thus, a sentence or paragraph standing *on its own* might be true or false, or even true and false, or true in more than one way, or false in more than one way.

I think it would be better to describe this as a *possibility* or *capacity* of language, rather than as a limitation.

There might be some aesthetic value in a series of open predicates that is independent of the truth set.

Some things to think about:

In terms of our responsibility toward others in our attempts to convince them, my own belief is that whether someone is speaking or writing truly or falsely depends *in part* on both parties involved, and that someone could use open predicates for benevolent *or* malicious purposes. The responsibility for the writer or speaker enters in with what he or she knows about the reader or hearer and his or her assumptions, as well as in the actual effect on the listener or reader.

If someone who *we think* is trying to take care of our car says, “I know you don’t want to think about this too closely,” and in the midst of our natural distraction takes several or several hundred dollars out of our wallet without our knowing, that might be an example of an open predicate being used in order *to steal*. Conversely perhaps, someone might use an open predicate to *prevent* the action of a thief, or even to **trap** him or her. But this takes us outside the content of this course, and there are some very important qualifications needed here which are beyond the scope of pure mathematics.

An open predicate also requires more *thought* on the part of the hearer or reader, and a *thoughtless* person might easily misunderstand it. It is sometimes good to slow down and think *very* carefully, even if an argument *seems* clear. Language as it is used in normal life is not always as *fixed* as it is in a purely mathematical context.