

Notes for Mathematical Reasoning, Writing, and Proof

By Nathan Hamlin

Definitions allow us to *clarify* the meaning of terms, but do not necessarily replace the terms.

Some mathematical entities are not words in the sense of visible marks on a page or sounds we can hear with our ears.

Logical relations often deal with *what* we can know and *how* we can know.

Others come to know mathematics through *proof* when

- 1) Our definitions are clear, precise, and effectually represent the entities referred to by the terms
- 2) Our reader can perceive the logical relations for him or herself
- 3) The logical relations effectively fall under a structure that is known to be sound reasoning: i.e. a known method of proof
- 4) We invoke, as necessary, theorems that are known, and which our reader knows to be known
- 5) We make the relevant algebraic or geometric steps sufficiently clear to our reader
- 6) Our writing is *clear* enough that others can perceive the definitions, theorems, logical relations, algebra, geometry, method(s) etc. as a *whole* in such a way
- 7) The language and symbols we use have a stable meaning in the social context of the readers that is sufficiently close to our own use of the symbols and language
- 8) We use the language and symbols well
- 9) Perhaps other conditions that are necessary (socially, physically, ...)

Mathematical *discovery* often occurs without all of these conditions in place; and the “burden of proof” is on the discoverer or his colleague or other interested parties to construct a proof, if a proof is possible.

That said, a good proof may leave room for doubt on the part of the reader, and the reader must not only give the proof his or her attention, but be open to the content.

Even in mathematics, there are some very controversial definitions, axioms, theorems, methods etc., as well as some that are probably a sham and known to be so.

A proof, then, is joint venture between someone who knows, or thinks he or she knows, and someone who *might* know. Thus, there is a “burden of proof” on the reader as well as well the writer, as well as social and physical conditions that either assist or perhaps obstruct the process of mutual reasoning.

This may not be an exhaustive account, but perhaps it is a good starting place.

