**Definition:** A set is a collection of objects. The objects belonging to the set are called the *elements* of the set.

Sets are commonly denoted with a capital letter, such as $S = \{1, 2, 3, 4\}$.

The set containing no elements is called the **empty set** (or **null set**) and is denoted by $\{\}$ or $\emptyset$. 
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Example. List all of the elements of each set using the listing method.

(a) The set $A$ of counting numbers between ten and twenty.

(b) The set $B$ of letters in the word “bumblebee.”

(c) $C = \{x \mid x \text{ is an even multiple of } 5 \text{ that is less than } 10\}$
Example. Denote each set by set-builder notation, using $x$ as the variable.

(a) The set $A$ of counting numbers between ten and twenty.

(b) The set $B$ of letters in the word “bumblebee.”

(c) $C = \{4, 8, 12\}$
Sets of Numbers and Cardinality

You should be familiar with the following special sets.

**Natural (counting) numbers:** \( \mathbb{N} = \{1, 2, 3, 4, \ldots\} \)

**Whole numbers:** \( W = \{0, 1, 2, 3, 4, \ldots\} \)

**Integers:** \( \mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} \)
Rational numbers: $\mathbb{Q} = \left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers, with } q \neq 0 \right\}$

Irrational numbers: $\{x \mid x \text{ cannot written as a quotient of integers}\}$.

Real numbers: $\mathbb{R} = \{x \mid x \text{ can be expressed as a decimal}\}$
To show that a particular item is an element of a set, we use the symbol $\in$.

The symbol $\notin$ shows that a particular item is *not* an element of a set.
Definition: The number of elements in a set is called the **cardinal number**, or **cardinality**, of the set.

This is denoted as $n(A)$, read “$n$ of $A$” or “the number of elements in set $A$.”
Example. Find the cardinal number of each set.

(a) The set $A$ of counting numbers between ten and twenty.

(b) The set $B$ of letters in the word “bumblebee.”

(c) $C = \{x|x$ is an even multiple of 5 that is less than 10$\}$
Subsets of a Set

**Definition:** Set $A$ is a *subset* of set $B$ if every element of $A$ is also an element of $B$. In symbols this is written as $A \subseteq B$. 
Example. Let $A = \{q, r, s\}$, $B = \{p, q, r, s, t\}$, and $C = \{p, q, z\}$. True or false:

(a) $A \subseteq B$

(b) $B \subseteq A$

(c) $C \subseteq B$
Equality of Sets

**Definition:** Set $A$ is equal to set $B$ provided the following two conditions are met:

1. Every element of $A$ is an element of $B$ ($A \subseteq B$)

   and

2. Every element of $B$ is an element of $A$ ($B \subseteq A$).
Example. State whether the sets in each pair are equal.

(a) \{a, b, c, d\} and \{a, c, d, b\}

(b) \{a, b, c, d\} and \{a, c, d, b, a\}

(c) \{5, 10, 15, 20\} and \{x| x is a multiple of 5 that is less than 20\}
Equivalence of Sets

**Definition:** Set $A$ is equivalent to set $B$ if $n(A) = n(B)$.

**Example.** State whether the sets in each pair are equivalent.

(a) \{a, b, c, d\} and \{a, c, d, b\}

(b) \{a, b, c, d\} and \{a, c, d, b, a\}