Section 2.1: Set Theory – Symbols, Terminology

Definition: A **set** is a collection of objects. The objects belonging to the set are called the **elements** of the set.

Sets are commonly denoted with a capital letter, such as $S = \{1, 2, 3, 4\}$.

The set containing no elements is called the **empty set** (or **null set**) and is denoted by { } or ∅.

Methods of Designating Sets	Example
1) A description in words	
2) Listing (roster) method	
3) Set-builder notation	

Example. List all of the elements of each set using the listing method.

(a) The set *A* of counting numbers between ten and twenty.

(b) The set *B* of letters in the word "bumblebee."

(c) $C = \{x \mid x \text{ is an even multiple of 5 that is less than 10}\}$

Example. Denote each set by set-builder notation, using x as the variable.

(a) The set A of counting numbers between ten and twenty.

(b) The set B of letters in the word “bumblebee.”

(c) $C = \{4, 8, 12\}$

Sets of Numbers and Cardinality

You should be familiar with the following special sets.

Natural (*counting*) **numbers**: $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

Whole numbers: $W = \{0, 1, 2, 3, 4, \ldots\}$

Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational numbers: $\mathbb{Q} = \left\{ \frac{p}{q} \middle| p \text{ and } q \text{ are integers, with } q \neq 0 \right\}$

Irrational numbers: $\{x \mid x$ cannot written as a quotient of integers$\}$.

Real numbers: $\mathbb{R} = \{x \mid x$ can be expressed as a decimal$\}$

To show that a particular item is an element of a set, we use the symbol $\in$.

The symbol $\notin$ shows that a particular item is *not* an element of a set.

Definition: The number of elements in a set is called the **cardinal number**, or **cardinality**, of the set.

This is denoted as ***n*(*A*),** read "*n* of *A*" or "the number of elements in set *A*."

Example. Find the cardinal number of each set.

(a) The set *A* of counting numbers between ten and twenty.

(b) The set *B* of letters in the word "bumblebee."

(c) $C = \{x | x$ is an even multiple of 5 that is less than $10\}$

Subsets of a Set

Definition: Set A is a *subset* of set B if every element of A is also an element of B. In symbols this is written as

$$A \subseteq B.$$

Example. Let $A = \{q, r, s\}$, $B = \{p, q, r, s, t\}$, and $C = \{p, q, z\}$. True or false:

(a) $A \subseteq B$

(b) $B \subseteq A$

(c) $C \subseteq B$

Equality of Sets

Definition: Set A is **equal** to set B provided the following two conditions are met:

1. Every element of A is an element of B ($A \subseteq B$)

   and

2. Every element of B is an element of A ($B \subseteq A$).

Example. State whether the sets in each pair are equal.

(a) {a, b, c, d} and {a, c, d, b}

(b) {a, b, c, d} and {a, c, d, b, a}

(c) {5, 10, 15, 20} and {x|x is a multiple of 5 that is less than 20}

Equivalence of Sets

Definition: Set *A* is **equivalent** to set *B* if $n(A) = n(B)$.

Example. State whether the sets in each pair are equivalent.

(a) {a, b, c, d} and {a, c, d, b}

(b) {a, b, c, d} and {a, c, d, b, a}