

MATH 416/516 MIDTERM EXAM 1

You may assume the availability of Uniform(0,1) RVs for all algorithms, but explicit formulas for transformation to other RVs must be given by you. Show all work and justify your answers.

1. (20 pts.) Determine an algorithm for the generation of a random variable with probability density function (**pdf**)

$$f(x) = \begin{cases} x/2 & 0 \leq x < 1 \\ 1/2 & 1 \leq x < 2 \\ (3-x)/2 & 2 \leq x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

2. (16 pts.) Determine an algorithm which uses the inverse transform method to generate a random variable with probability mass function (**pmf**) given by

$$P\{X = j\} = \binom{3}{j} \left(\frac{1}{3}\right)^j \left(\frac{2}{3}\right)^{3-j}, \quad j = 0, 1, 2, 3.$$

3. (12 pts.) Determine an algorithm which uses the inverse transform method to generate a random variable with cumulative distribution function (**cdf**)

$$F(x) = \left(1 - \frac{2 \cos^{-1}(x)}{\pi}\right)^2, \quad 0 \leq x \leq 1$$

4. (18 pts.) Determine an algorithm which uses the composition method to generate a random variable with cumulative distribution function (**cdf**)

$$F(x) = \frac{x^2}{4} + \frac{x^3}{4} + \frac{e^x}{2(e-1)}, \quad 0 \leq x \leq 1$$

5. (16 pts.) Orders arrive at a lumber company according to a homogeneous Poisson process with rate 6 per hour, and each order generates an amount of revenue that is exponentially distributed with mean \$200. Write an algorithm to simulate the amount of total revenue for the company for an 8 hour day.

6. (18 pts.) Determine an acceptance-rejection algorithm for the generation of a random variable with probability density function (**pdf**)

$$f(x) = 12x(1 - x)^2, \quad 0 \leq x \leq 1,$$

using $g(x) = 1$.