TOTAL DIFFERENTIALS and APPROXIMATIONS

Total Differentials for Two Variables
for a function \( z = f(x, y) \).

- **Definition:** the total differential for \( f \) is
  \[ dz = df = f_x(x, y)dx + f_y(x, y)dy \]

- **Approximations:** given small values for \( \Delta x \) and \( \Delta y \),
  \[ \Delta z = \Delta f = f_x(x, y)\Delta x + f_y(x, y)\Delta y, \]
  and
  \[ f(x+\Delta x, y+\Delta y) \approx f(x, y) + f_x(x, y)\Delta x + f_y(x, y)\Delta y. \]

- **Tangent Plane to surface at \((a, b)\):**
  \[ z = f(a, b) + (x - a)f_x(a, b) + (y - b)f_y(a, b). \]

Total Differentials for Three Variables
for function \( w = f(x, y, z) \).

- **Definition:** the total differential for \( f \) is
  \[ dw = df = f_x(x, y, z)dx + f_y(x, y, z)dy + f_z(x, y, z)dz \]

- **Approximations:** given small values for \( \Delta x \) and \( \Delta y \),
  \[ \Delta w = \Delta f = f_x(x, y, z)\Delta x + f_y(x, y, z)\Delta y + f_z(x, y, z)\Delta z, \]
  and
  \[ f(x + \Delta x, y + \Delta y, z + \Delta z) \approx f(x, y, z)
  + f_x(x, y, z)\Delta x + f_y(x, y, z)\Delta y + f_z(x, y, z)\Delta z. \]

- **Tangent Plane to surface at \((a, b, c)\):**
  \[ w = f(a, b, c) + (x - a)f_x(a, b, c) + (y - b)f_y(a, b, c)
  + (z - c)f_z(a, b, c). \]
Total Differentials Examples

• Determine $dV$ for beer can with volume $V = \pi r^2 h$ when $(r, h) = (1, 5)$; sensitivity for $dr$ versus $dh$?

• If $z = f(x, y) = x \ln(y)$, find $dz$, and approximate $1.04 \ln(.95)$. 

TOTAL DIFFs and APPROXs CONT.
ezsurf('x*log(y)',[-3 4 .5 2.5]); hold on
ezsurf('y-1',[-3 4 .5 2.5]), view([135 15])
• If \( w = \frac{5x^2 + y^2}{z + 1} \), find \( dw \) and approximate \( w(-1.98, .97, 1.03) \).

• If oxygen consumption is \( m = 2.5(T - F)w^{-0.67} \), for a mammal, find \( dm \) when \( T : 38 \rightarrow 36, F : 12 \rightarrow 13, w : 30 \rightarrow 31 \).
TOTAL DIFFs and APPROXs CONT.

• If \( f(x, y) = x^2 + y^2 \); find total differential, tangent plane at \((-1, 1)\)?

Using Matlab

\[
ezsurf('x^2+y^2'); \text{ hold on} \\
ezsurf('2-2*(x+1)+2*(y-1)'), \text{view([135 10])}
\]
TOTAL DIFFs and APPROXs EXs

- If \( f(x, y) = (x^2 + 4y^2)^{1/3} \); total differential at \((2, -1)\)?, tangent plane at \((2, -1)\)?

Using Matlab

\[
ezsurf('(x^2+4*y^2)^(1/3)'); \text{ hold on} \\
ezsurf('2+(x-2)/3-2(y+1)/3')
\]