EXAM 1, Math 273 fall 2015

Name: ____________________________ ID#: __________________

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Do any 6 of these 8 problems. You must ex out or leave blank the work space for the two problems you do not want graded. The numbers in the [] tell what each problem or part is worth. Unsimplified answers are generally okay, though we might take a point for leaving things like \( \cos(\pi) \) or \( e^0 \) or \( \ln(1) \). Show work or explanation on everything.

[15] 1. Find the equation of the plane that contains both of the intersecting lines \( \vec{r}_1(t) = (2 + t, 6 - 3t, 1 + 4t) \) and \( \vec{r}_2(s) = (2 + s, 6 + 2s, 1 + 3s) \).

\[
\langle 1,-3,4 \rangle \times \langle 1,2,3 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \langle -17, 1, 5 \rangle.
\]

So plane eqn. is

\[
-17(x-2) + (y-6) + 5(z-1) = 0
\]

or

\[
-17x + y + 5z = -23
\]

or

\[
17x - y - 5z = 23
\]

[15] 2. Find parametric equations for the line that contains the point \((4, 3, -1)\) and is perpendicular to the plane \(7x - 2y + z = 10\).

\[
\begin{cases} 
  x = 4 + 7t \\
  y = 3 - 2t \\
  z = -1 + t
\end{cases}
\]

or

\[
\vec{r}(t) = \langle 4 + 7t, 3 - 2t, -1 + t \rangle.
\]
[15] 3. For the curve \( \vec{r}(t) = (\sqrt{t}, 4-t) \), do the following:

5) a) Find \( \vec{r}'(t) \).

\[
\vec{r}'(t) = \left\langle \frac{1}{2\sqrt{t}}, -1 \right\rangle
\]

[10] b) Find the \( t \) value for which \( \vec{r}(t) \) and \( \vec{r}'(t) \) are orthogonal. That is, find \( t \) such that \( \vec{r}(t) \cdot \vec{r}'(t) = 0 \).

\[
\vec{r} \cdot \vec{r}' = \left\langle \sqrt{t}, 4-t \right\rangle \cdot \left\langle \frac{1}{2\sqrt{t}}, -1 \right\rangle = \frac{1}{2} - 4 + t = t - \frac{7}{2}.
\]
This is zero when \( t = \frac{7}{2} \).

[15] 4. Suppose a particle’s acceleration is \( \vec{a}(t) = (e^{-t}, 5, t^3) \), and the particle’s initial velocity is \( \vec{v}(0) = (0, 2, 1) \).

[10] a) Find the particle’s velocity function \( \vec{v}(t) \).

\[
\vec{v} = \int \vec{a} \, dt = \left\langle -e^{-t} + A, 5t + B, \frac{1}{4} t^4 + C \right\rangle.
\]

\[
\vec{v}(0) = (0, 2, 1) \Rightarrow \left\langle -A + B, 0, C \right\rangle = (0, 2, 1) \Rightarrow A = 1, B = 2, C = 1.
\]

So \( \vec{v}(t) = \left\langle 1 - e^{-t}, 5t + 2, \frac{1}{4} t^4 + 1 \right\rangle \).

[5] b) Set up an integral whose value would give the distance travelled by the particle from \( t = 0 \) to \( t = 1 \).

\[
L = \int_0^1 |\vec{v}(t)| \, dt = \int_0^1 \sqrt{(1-e^{-t})^2 + (5t+2)^2 + \left(\frac{1}{4} t^4 + 1\right)^2} \, dt
\]
5. For the surface \( \frac{x^2}{4} + \frac{z^2}{9} = y + 1 \), do the following:

(a) Below, the surface's trace in the xy-plane has been found and graphed. Find and graph the surface's traces in the xz- and yz-planes.

xy trace: set \( z = 0 \), get \( \frac{x^2}{4} = y + 1 \), so \( y = \frac{x^2}{4} - 1 \).

xz trace: set \( y = 0 \), get \( \frac{x^2}{4} + \frac{z^2}{9} = 1 \).

yz trace: set \( x = 0 \), get \( y = \frac{z^2}{9} - 1 \).

(b) Which graph below best represents this surface? (circle your answer)

6. For the function \( f(x, y) = \sqrt{x + y^2} \), graph the level curves (a.k.a. contours) \( f(x, y) = 0 \), \( f(x, y) = 1 \), \( f(x, y) = 2 \), \( f(x, y) = 3 \) in the window provided. Label each curve with its z-level.

\[
\sqrt{x + y^2} = 0 \Rightarrow x = -y^2
\]
\[
\sqrt{x + y^2} = 1 \Rightarrow x = 1 - y^2
\]
\[
\sqrt{x + y^2} = 2 \Rightarrow x = 4 - y^2
\]
\[
\sqrt{x + y^2} = 3 \Rightarrow x = 9 - y^2
\]
Let function $h$ be defined like this: $h(x, y) = \begin{cases} \frac{5xy}{x^2+y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0) \end{cases}$

Prove that $\lim_{(x,y) \to (0,0)} \frac{5xy}{x^2+2y^2}$ does not exist.

Along $y$-axis, get $\frac{5xy}{x^2+2y^2} = \frac{5\cdot0\cdot y}{0^2+2y^2} = \frac{0}{2y^2} \to 0$.

Along $y=x$, get $\frac{5xy}{x^2+2y^2} = \frac{5x^2}{x^2+2x^2} = \frac{5x^2}{3x^2} = \frac{5}{3} \to \frac{5}{3}$.

So by the path test, limit DNE.

b) At which points in $\mathbb{R}^2$ is $h$ continuous? At which points in $\mathbb{R}^2$ is $h$ discontinuous? Explain.

$\frac{5xy}{x^2+2y^2}$ is continuous at all points other than $(0,0)$, and $h$ coincides with this function at these points, so $h$ is cont. at all $(x,y) \neq (0,0)$.

$h$ is discontinuous at $(0,0)$ because $\lim_{(x,y) \to (0,0)} h(x,y)$ is the limit above, which DNE.

Find $f_x(x, y)$ and $f_y(x, y)$ for the function $f(x, y) = x^3y^2 + 6y - \frac{y}{x^2}$.

\[
\frac{df}{dx} = \frac{3x^2y^2 + 3y}{x^2}
\]

\[
\frac{df}{dy} = 2x^3y + 6 - \frac{1}{x^2}
\]