Math 273 EXAM 1 Name: ____________________________

DIRECTIONS: This exam has 6 questions totalling 80 points, but it will be weighted as a 100 point exam in the grade book. Numbers in the [ ] tell what each problem is worth. Unsimplified answers are generally okay, though we might take a point for leaving things like \( \cos(\pi) \) or \( e^0 \) or \( \ln(1) \). Show work or explanation on everything. Back page is blank (extra work space).

1. [8,12] For the space curve \( \mathbf{r}(t) = \langle \sqrt{t+1} , \ln(3t+1) , 2e^{-t} \rangle \), do the following:

a) Assuming \( \mathbf{r}(t) \) represents a particle’s position, find the particle’s velocity and acceleration vectors at the point where \( t = 1 \).

\[
\mathbf{r}'(1) = \langle \frac{1}{2\sqrt{2}} , \frac{3}{3(1)} , -2e^{-1} \rangle, \quad \mathbf{r}''(1) = \langle -\frac{1}{4(3t+1)^{3/2}} , \frac{-9}{(3t+1)^2} , 2e^{-1} \rangle.
\]

velocity at \( t=1 \) is \( \mathbf{r}'(1) = \langle \frac{1}{2\sqrt{2}} , \frac{3}{3} , \frac{-2}{e} \rangle \)

accel. at \( t=1 \) is \( \mathbf{r}''(1) = \langle -\frac{1}{8\sqrt{2}} , \frac{-9}{16} , \frac{2}{e} \rangle \).

b) Find parametric equations for the line that is tangent to this curve at the point \( (1, 0, 2) \).

point \( (1,0,2) \) corresponds to \( t=0 \).

Direction vector is \( \mathbf{r}'(0) = \langle \frac{1}{2} , 3 , -2 \rangle \). So:

\[
\begin{align*}
x &= 1 - \frac{1}{2} t \\
y &= 0 + 3t \\
z &= 2 - 2t
\end{align*}
\]

2. [12] If \( v = x^2y^3\sin(4x-z^2) \), find the partial derivatives \( v_x, v_y \) and \( v_{yz} \).

\[
\frac{dv}{dx} = 2xy^3\sin(4x-z^2) + 4x^2y^3\cos(4x-z^2)
\]

\[
\frac{dv}{dy} = 3x^2y^2\sin(4x-z^2)
\]

\[
\frac{d^2v}{dxdy} = \frac{d}{d\varepsilon} (above) = 3x^2y^2(-2z\cos(4x-z^2)) = -6x^2y^2z\cos(4x-z^2)
\]
3. Sketch the surface \( z^2 - 4y + 4x^2 = 0 \) as well as you can. The use of traces is recommended.

Equivalent to \( y = \frac{z^2}{4} + x^2 \). Traces \( y = k \) are ellipses with shape

Trace \( z = 0 \) is \( y = x^2 \).

Trace \( x = 0 \) is \( y = \frac{1}{4} z^2 \). So:

4. Prove that \( \lim_{(x,y) \to (0,0)} \frac{xy + y^4}{3x^2 + y^2} \) does not exist.

As \( (x,y) \to (0,0) \) along the x-axis,

\[
\frac{xy + y^4}{3x^2 + y^2} = \frac{0}{3x^2} \to 0.
\]

As \( (x,y) \to (0,0) \) along the line \( y = x \),

\[
\frac{xy + y^4}{3x^2 + y^2} = \frac{x^2 + x^4}{3x^2 + x^2} = \frac{x^2(1+x^2)}{4x^2}
\]

\[
= \frac{1+x^2}{4} \to \frac{1}{4}.
\]

So two target levels along different paths, so this limit DNE.
5. [12] Draw a contour map of the function \( f(x, y) = \frac{x^2 + 1}{y - 1} \), showing level curves for \( z \)-levels of \(-2, -1, \frac{1}{2}, 1, 2\). Label each contour with its \( z \)-level.

\[
\frac{x^2 + 1}{y - 1} = k \implies y - 1 = \frac{k}{x^2 + 1} \implies y = \frac{k}{x^2} + \left( \frac{1}{x^2} + 1 \right). \quad \text{Parabolas.}
\]

\[
\begin{array}{c|c}
 k & \text{trace/contour} \\
-2 & y = -\frac{x^2}{2} + \frac{1}{2} \\
-1 & y = -x^2 \\
\frac{1}{2} & y = -2x^2 - 1 \\
\frac{1}{2} & y = 2x^2 + 3 \\
1 & y = x^2 + 2 \\
2 & y = \frac{1}{2}x^2 + \frac{3}{2}
\end{array}
\]

Note that \( y \) cannot equal 1.

6. [12] Find the equation of the plane that is tangent to the surface \( z = \ln(y^2 - x) + \frac{2y}{x^2} \) at the point where \((x, y) = (3, 2)\).

\[
\frac{\partial z}{\partial x} = -\frac{1}{y^2 - x} - \frac{4y}{x^3}, \quad \frac{\partial z}{\partial y} = \frac{2y}{y^2 - x} + \frac{2}{x^2}
\]

\[
\frac{\partial z}{\partial x}(3, 2) = -1 - \frac{8}{27} = -\frac{35}{27}, \quad \frac{\partial z}{\partial y}(3, 2) = 4 + \frac{2}{9} = \frac{38}{9}, \quad z(3, 2) = \ln(1) + \frac{4}{9} = \frac{4}{9}.
\]

So \( 1 \frac{1}{2} \)

\[
\boxed{z = \frac{4}{9} - \frac{35}{27}(x - 3) + \frac{38}{9}(y - 2)} = -\frac{35}{27}x + \frac{31}{9}y - \frac{33}{9}.
\]