Optimal Selection of Habitat Reserves

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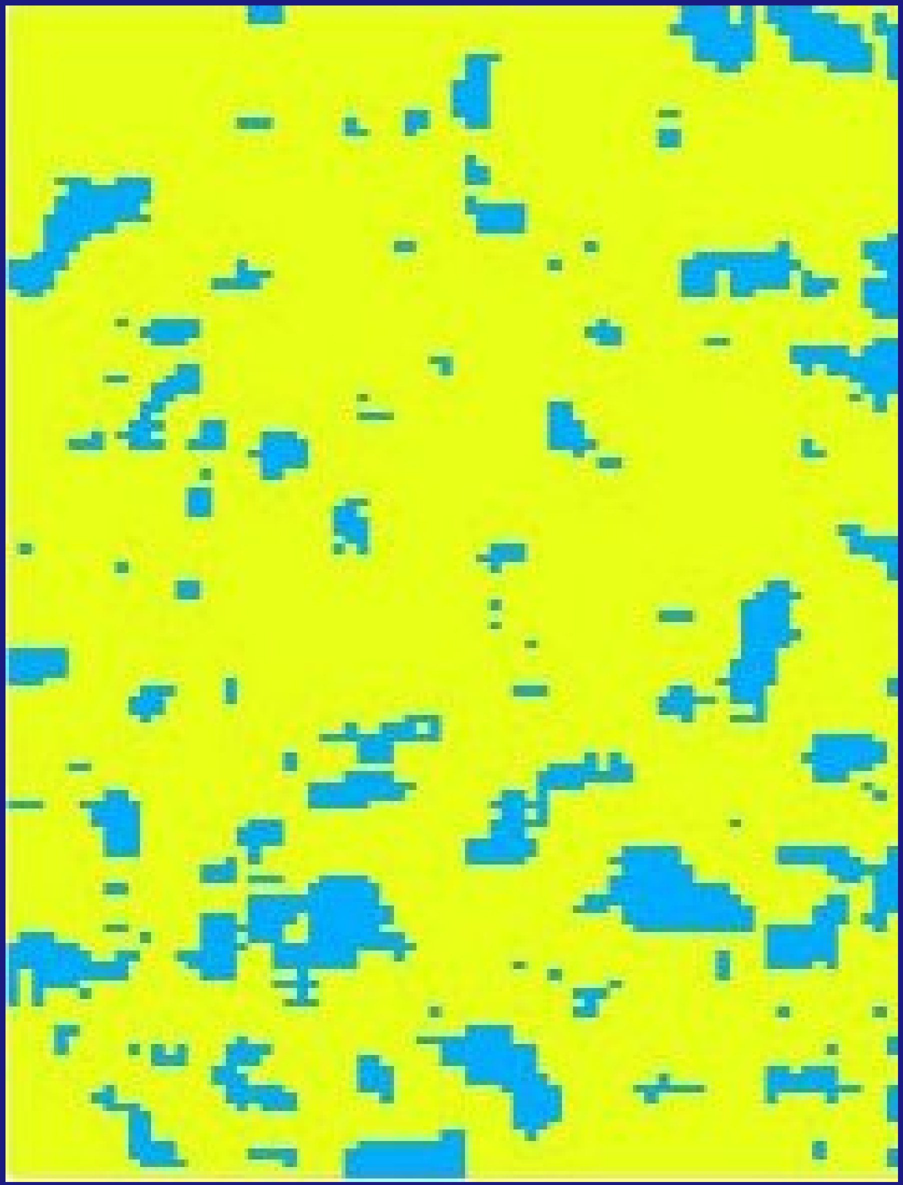
The Problem

- Habitat reserve design for territorial disperser
  e.g., the Northern Spotted Owl (NSO)
- Binary Habitat
  Green = Suitable
  Beige = Unsuitable
- Declining population
- Which habitat areas will best augment an existing reserve area?

NSO Habitat Landscape



Life Cycle Parameters

- Territorial species
- Home territory $\approx$ 900 hectares
- Adult survival rate $\approx$ 0.91
- Birth rate and juvenile survival rate combined $\approx$ 0.20
- Juvenile dispersal process - sensitive to fragmentation
- Mean dispersal distance: 15km

Dynamic Population Model

- Model habitat as array of cells, each cell the size of a home territory

- Habitat cells are either suitable or unsuitable
  $H_{ij} = 1$ or 0

- Randomly seed adults (nesting pairs)

- Model several cycles of mortality, birth pulse, dispersal, until population reaches equilibrium

- Compute average occupancy rate for each cell as a measure of the value of the cell

Limitations of Dynamic Model

- Captures species life history dynamics
- Not useful for constrained optimization
- Need an easy to evaluate function that measures the "suitability" of a habitat for the survival of the species

Index Function Modeling Problem

- Develop an explicit landscape function (HCI) that measures habitat “values” similar to the dynamic population model (i.e., calculate occupancy rates)
- Use the explicit function to formulate constrained optimization problems in resource management

The Habitat Connectivity Index (HCI)

- The HCI is calculated as a “colonization probability”
- Each suitable cell has an independent probability of colonizing the focal cell
- The probability of colonization is a function of distance

Colonization

- Probability that a dispersing juvenile will claim a specific habitat cell
- If the location of the natal cell is (k, l) then the probability that a dispersal attempts to claim cell $(k+i, l+j)$ is p_{ij} (depends on distance d_{ij}, but independent of k, l)
- The probability that at least one neighboring cell manages to colonize cell (i, j) is given by

$$1 - \prod_{k,l} (1 - p_{kl} H_{i-k,j-l})$$

Colonization

- HCI for cell (i, j) is the product of the habitat value for that cell, H_{ij}, and the probability that at least one (other) cell will colonize (i, j):

$$HCI_{ij} = H_{ij}\left[1 - \prod_{k,l}(1 - p_{kl}H_{i-k,j-l})\right]$$

- Recall $H_{i-k,j-l} = 0$ or 1,

$$\ln(1 - p_{kl}H_{i-k,j-l}) = H_{i-k,j-l}\ln(1 - p_{kl})$$

- Let $L_{kl} = \ln(1 - p_{kl})$ (*log-dispersal propensity*)

Habitat Connectivity Index

$$HCI_{ij} = H_{ij}\left(1 - e^{\sum_k \sum_l (H_{i-k,j-l} L_{kl})}\right)$$

- L_{kl} is obtained through the tuning process (regression)
- For the entire landscape, $HCI = \sum_{\text{all } i,j} HCI_{ij}$
- Highly nonlinear

Example Calculation

0	1	1	1	0	1
1	1	1	1	0	0
1	1	1	1	0	0
1	1	1	0	0	0
1	1	1	0	0	0
0	1	1	1	0	0

$L =$

-0.3	-0.5	-0.3
-0.5	0	-0.5
-0.3	-0.5	-0.3

$$HCI_{ij} = 1 \times \left(1 - e^{-0.3-0.5-0.5-0.3}\right) \approx 0.8$$

The Motivating Problem

- Want to determine which areas of the habitat to maintain, and which ones could be cleared or utilized for other activities (e.g., logging)

- make sure that the species will survive in the altered habitat

- Develop a constrained optimization problem

$$\begin{aligned} \max \quad & HCI(x_1, x_2, \ldots, x_n) \\ \text{s.t.} \quad & c_1x_1 + c_2x_2 + \ldots + c_nx_n \quad \leq \; G \\ & x_j \in \{0,1\} \text{ for } j = 1, \ldots, n \end{aligned}$$

- e.g., can maintain as green *at most* 15 candidate squares

$$\Rightarrow \quad c_1, \ldots, c_n = 1, \quad G = 15$$

Quadratic Model

- $HCI = \sum_i \sum_j H_{ij} \left[1 - \text{exp}\left(\sum_k \sum_l H_{i-k,j-l} L_{kl} \right)\right]$
- need to use (nicer) approximations of $HCI_{ij} = H_{ij}(1 - e^{t_{ij}})$

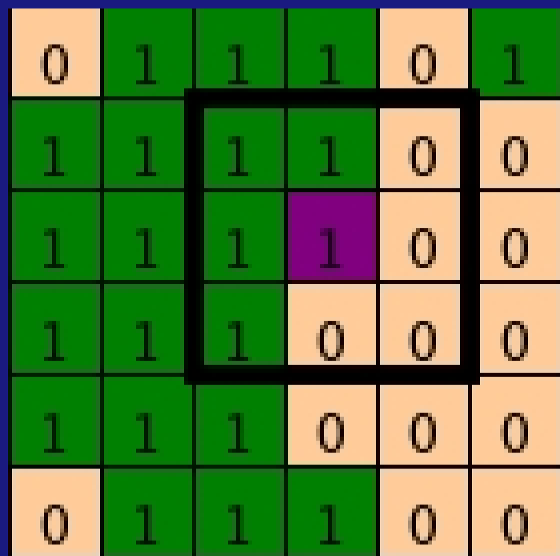
$$\text{where} \quad t_{ij} = \sum_k \sum_l (H_{i-k,j-l} L_{kl})$$

- notice that $e^t \approx 1 + t$ for small t, so define

$$QHCI_{ij} = -H_{ij} \sum_k \sum_l (H_{i-k,j-l} L_{kl})$$

- $QHCI = \sum_{i,j} QHCI_{ij}$ is positive semidefinite, so *easier* to solve the optimization problem

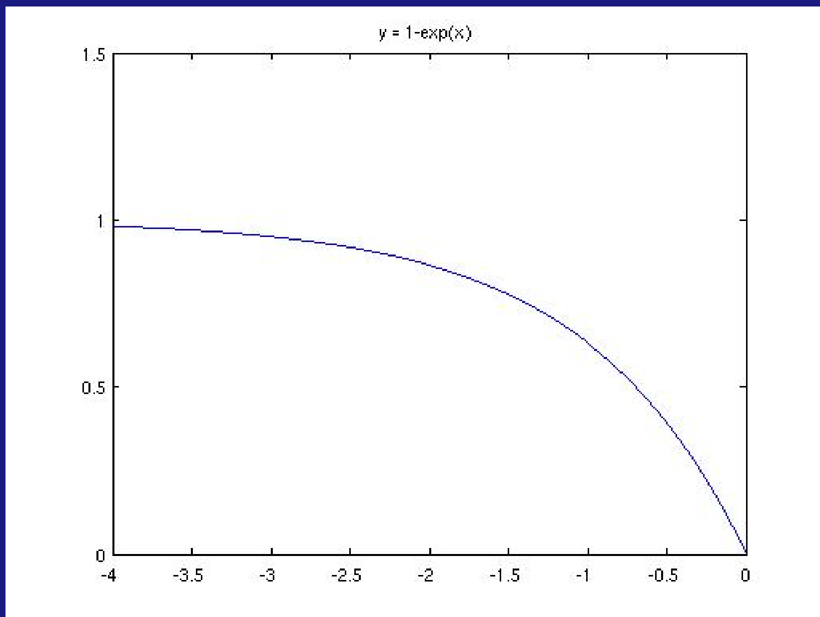
Example Calculation – Quadratic Model

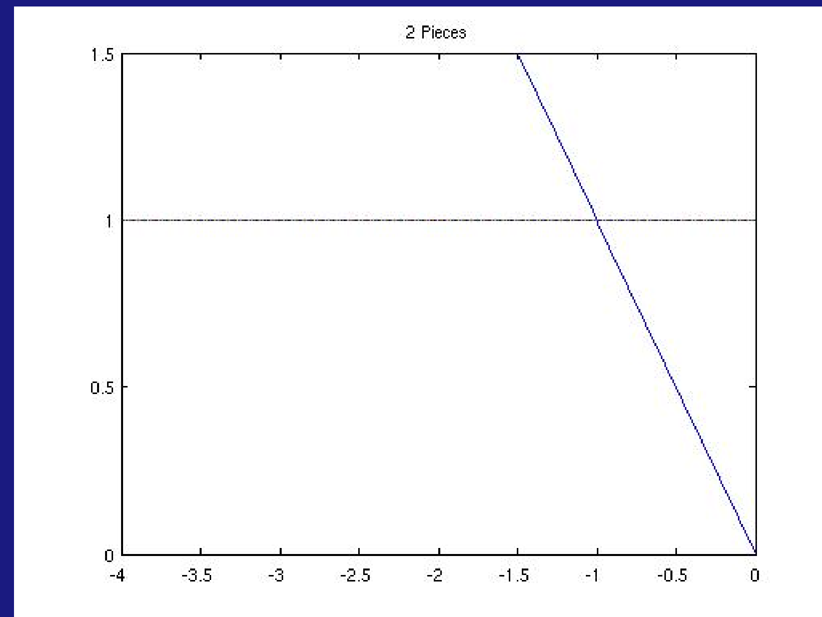


$$L = \begin{array}{|c|c|c|} \hline -0.3 & -0.5 & -0.3 \\ \hline -0.5 & 0 & -0.5 \\ \hline -0.3 & -0.5 & -0.3 \\ \hline \end{array}$$

$$QHCI_{ij} = -1 \times (-0.3 - 0.5 - 0.5 - 0.3) = 1.6$$

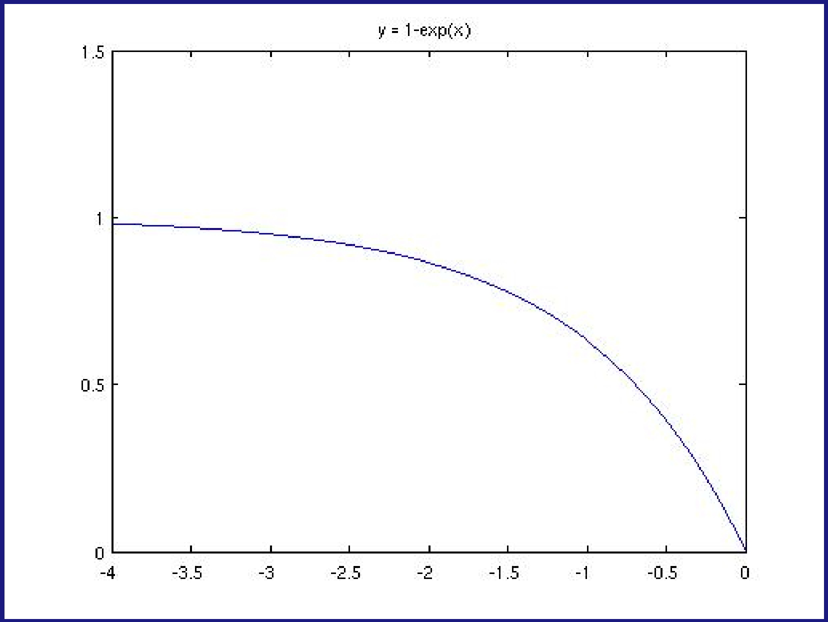
Graph of HCI_{ij} vs $QHCI_{ij}$

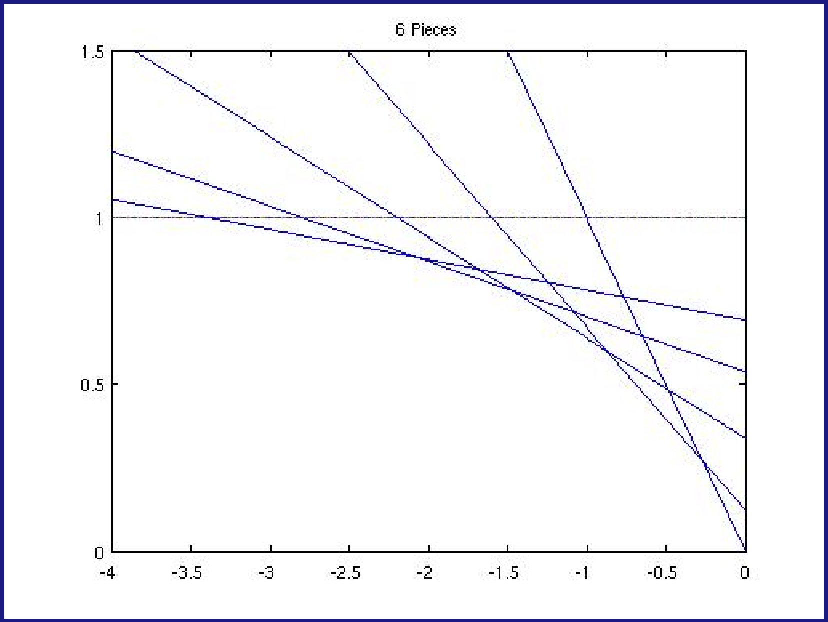




- $QHCI_{ij}$ is not a good approximation of HCI_{ij}
- note that $1 - e^t \leq -t$ and also $1 - e^t \leq 1$

Linearized Model





- use *several* linear pieces to represent $1 - e^{t_{ij}}$

Linearized Model

- Let y_{ij} *represent* $1 - e^{t_{ij}}$. To use t pieces, we write

$$\begin{aligned} y_{ij} &\leq P_k(t_{ij}),\ k = 1, \ldots, t,\ \text{where} \\ P_1(t_{ij}) &= 1 \\ P_2(t_{ij}) &= -t_{ij} \\ P_3(t_{ij}) &= -0.5\, t_{ij} + 0.35\ \text{(say)} \\ &\cdots \end{aligned}$$

- define $PLQHCI_{ij} = H_{ij} \times y_{ij}$, and add these constraints
- need to further linearize quadratic terms $(x_{ij} \times y_{ij})$ ...

Fully Linearized Model

- let z_{ij} *represent* the quadratic term $x_{ij} \times y_{ij}$
- define $PLHCI_{ij}$ by replacing $x_{ij} \times y_{ij}$ with z_{ij} in $PLQHCI_{ij}$
- add the constraints

$$\begin{aligned} z_{ij} &\leq y_{ij} + (1 - x_{ij}) \\ z_{ij} &\leq x_{ij} \end{aligned}$$

recall: x_{ij} is 0 or 1

- $PLHCI = \sum_{i,j} PLHCI_{ij}$

Computational Results

- 125×75 landscape map, 51×51 dispersal range
- Vary # candidate habitat cells (n) and max # of candidate cells that can be kept green (G)

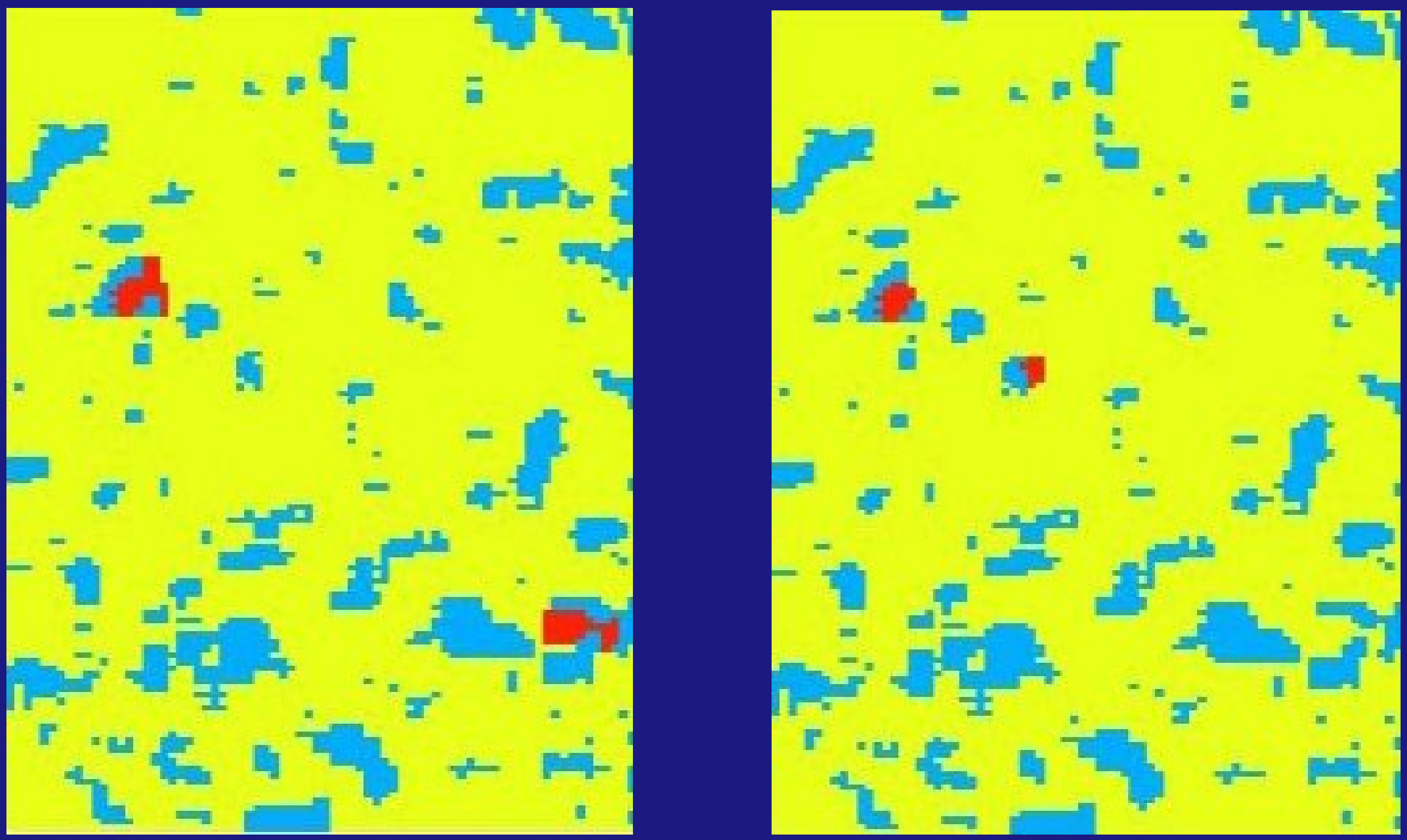
$$\begin{array}{lll} \max & HCI(x_1, x_2, \ldots, x_n) & \\ \text{s.t.} & x_1 + x_2 + \ldots + x_n & \leq \quad G \\ & x_j \in \{0, 1\} & \text{for } j = 1, \ldots, n \end{array}$$

with $QHCI$ and $PLHCI$ replacing HCI

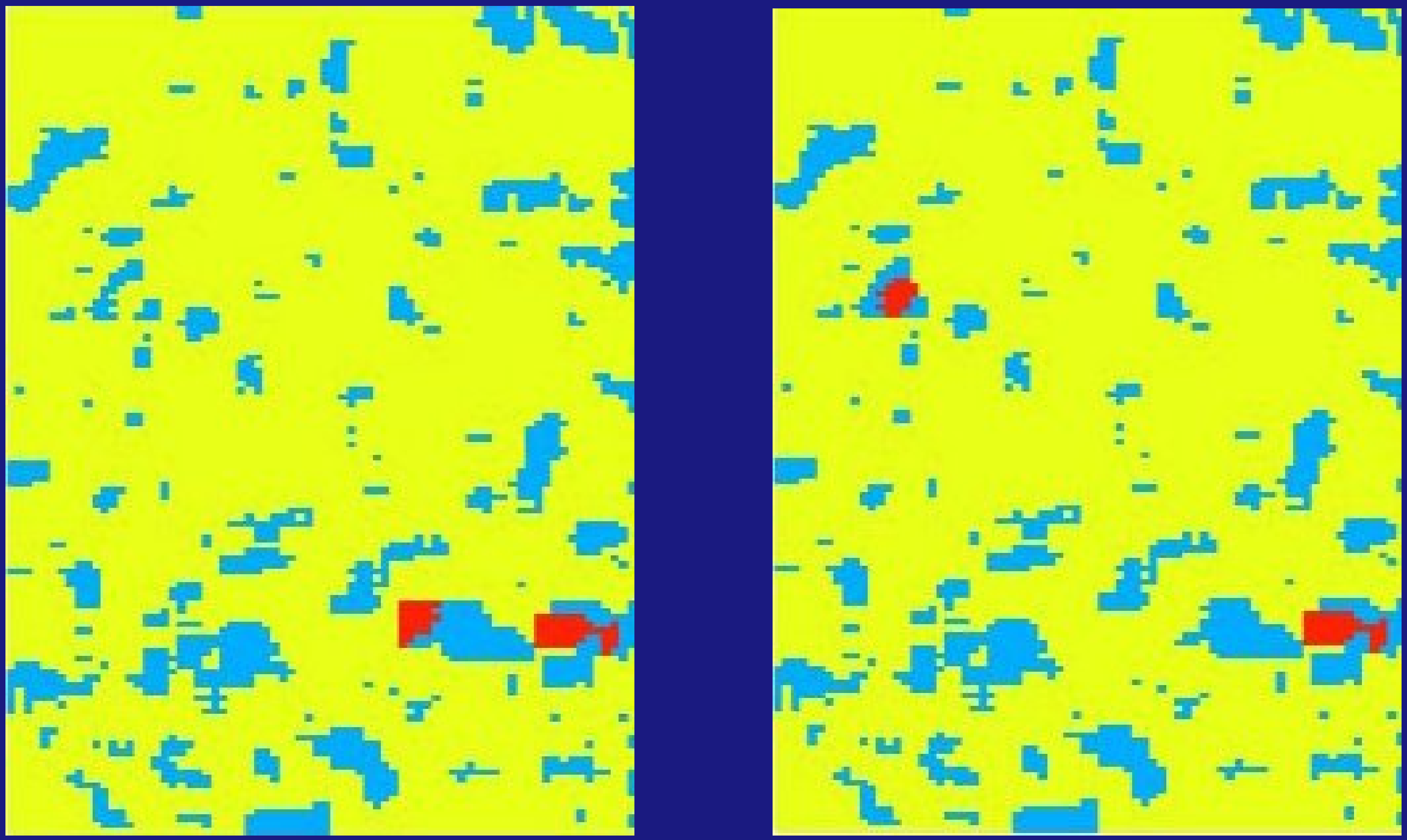
- Using AMPL-CPLEX software (state-of-the-art)
- Note running times and # branch-and-bound (BB) nodes

Candidate Habitats

- tried many different configurations of candidate habitats (red cells)



Candidate Habitats



Quadratic Model – varying n

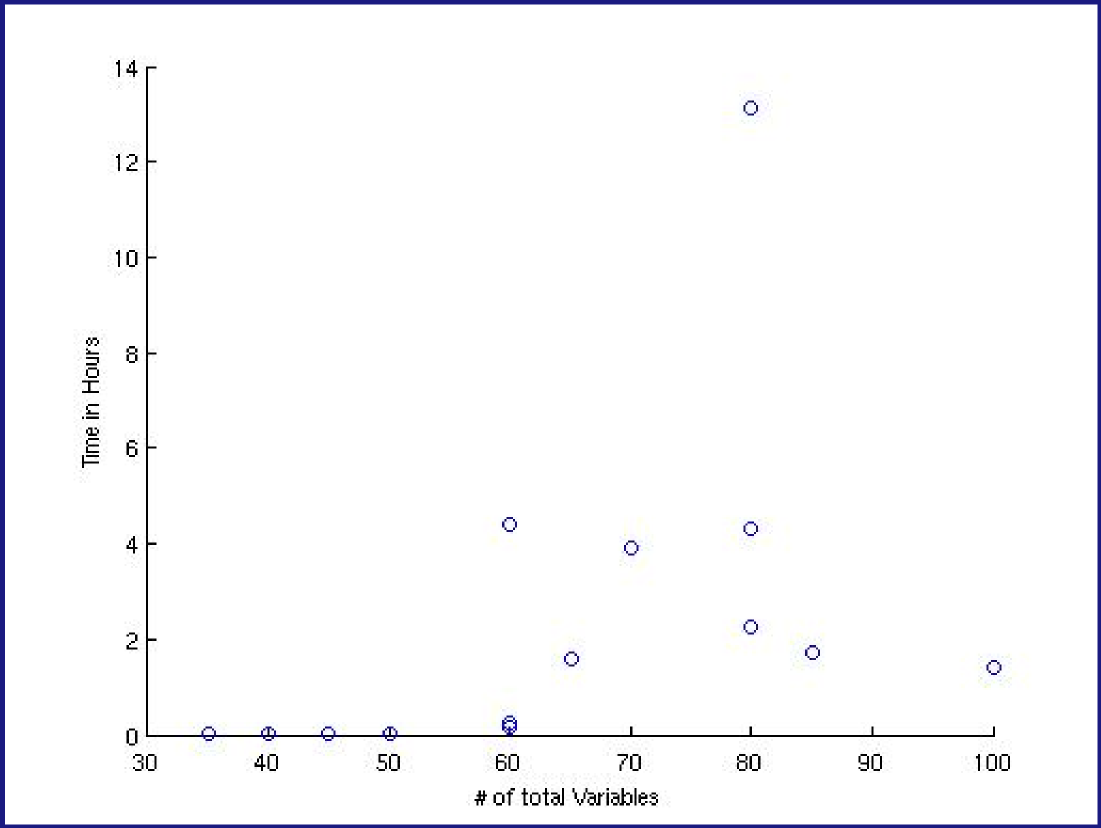


Figure 1: Run times (in hours) of the quadratic model for varying total # cells in the two (red) patches, with $G = 25$.

Quadratic Model – varying G

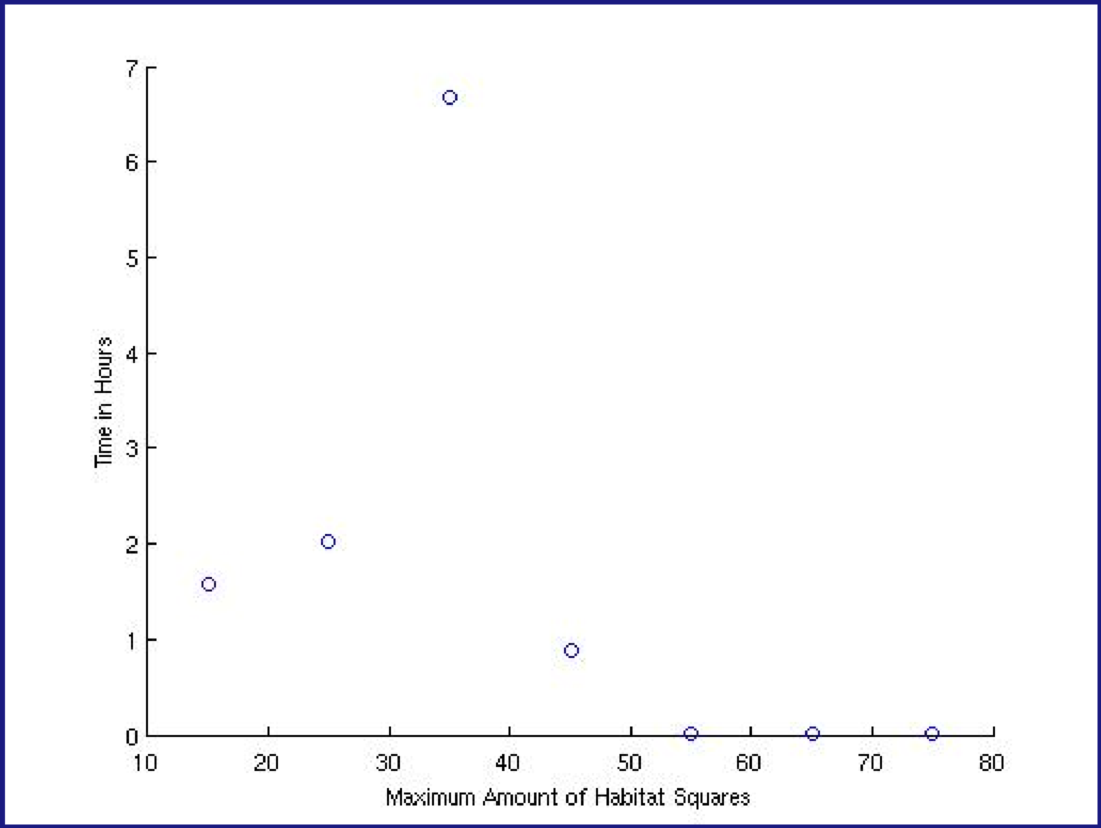
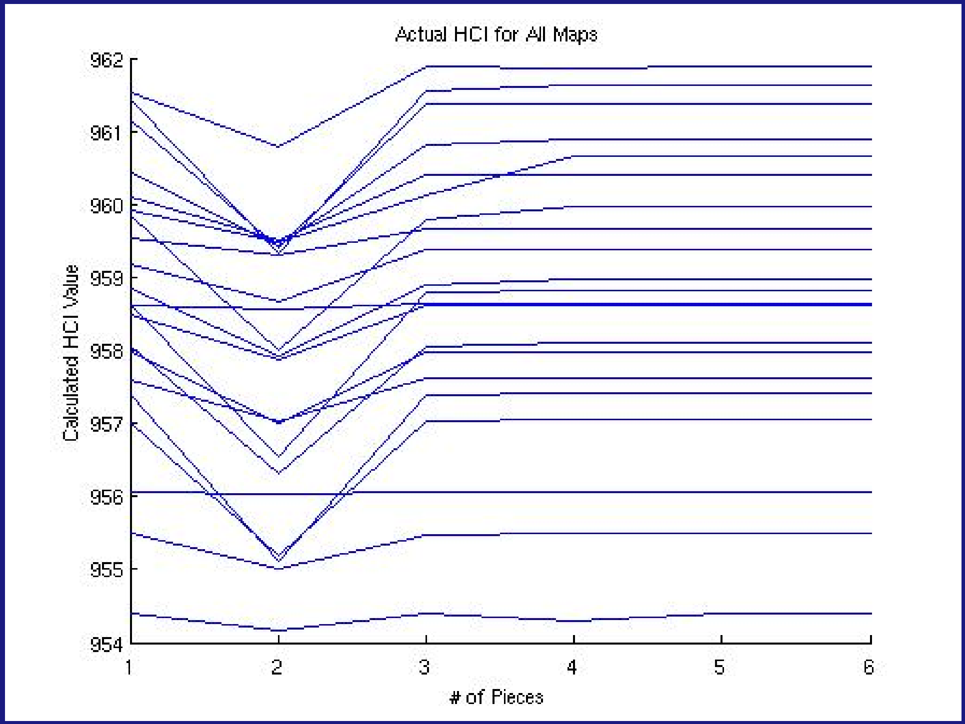


Figure 2: Run times (in hours) of the quadratic model for varying maximum # candidate cells allowed to be green, with $n = 80$.

How many pieces are needed?



- evaluate HCI at optimal solutions obtained using $PLHCI$ with various numbers of linear pieces
- 3 Pieces are sufficient

Linearized Model (3 Pieces) – varying n

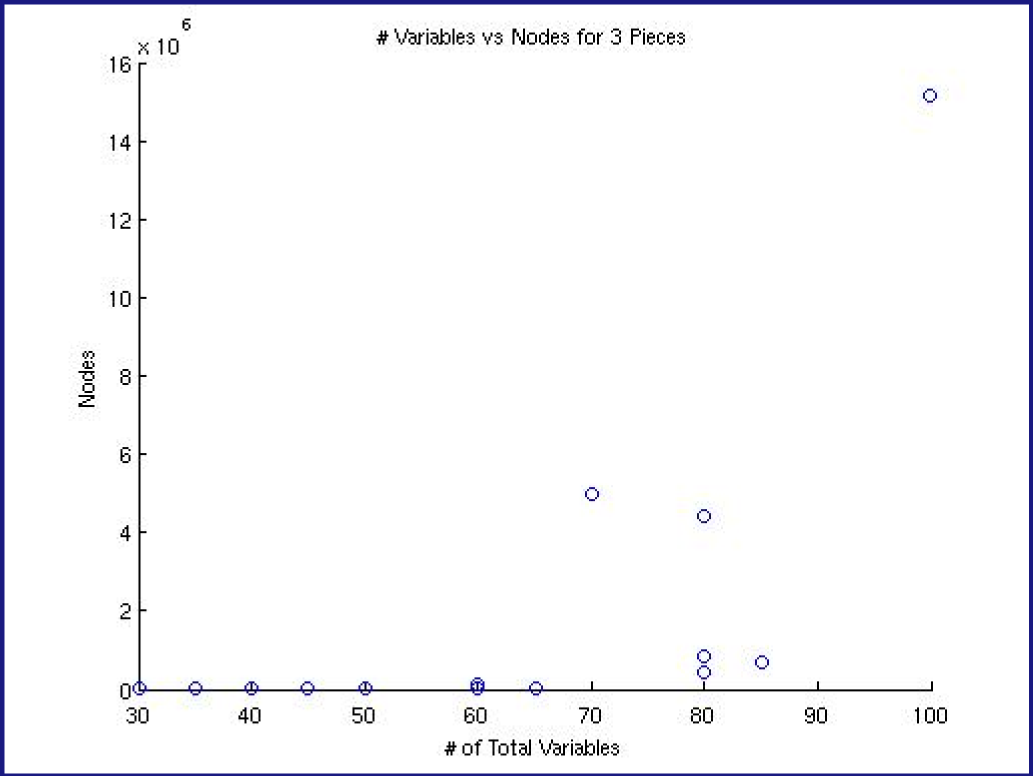


Figure 3: # BB nodes for solving the linearized model for varying total # cells in the two (red) patches, with $G = 25$.

Note: # constraints is large for large n

Linearized Model (3 Pieces) – varying G

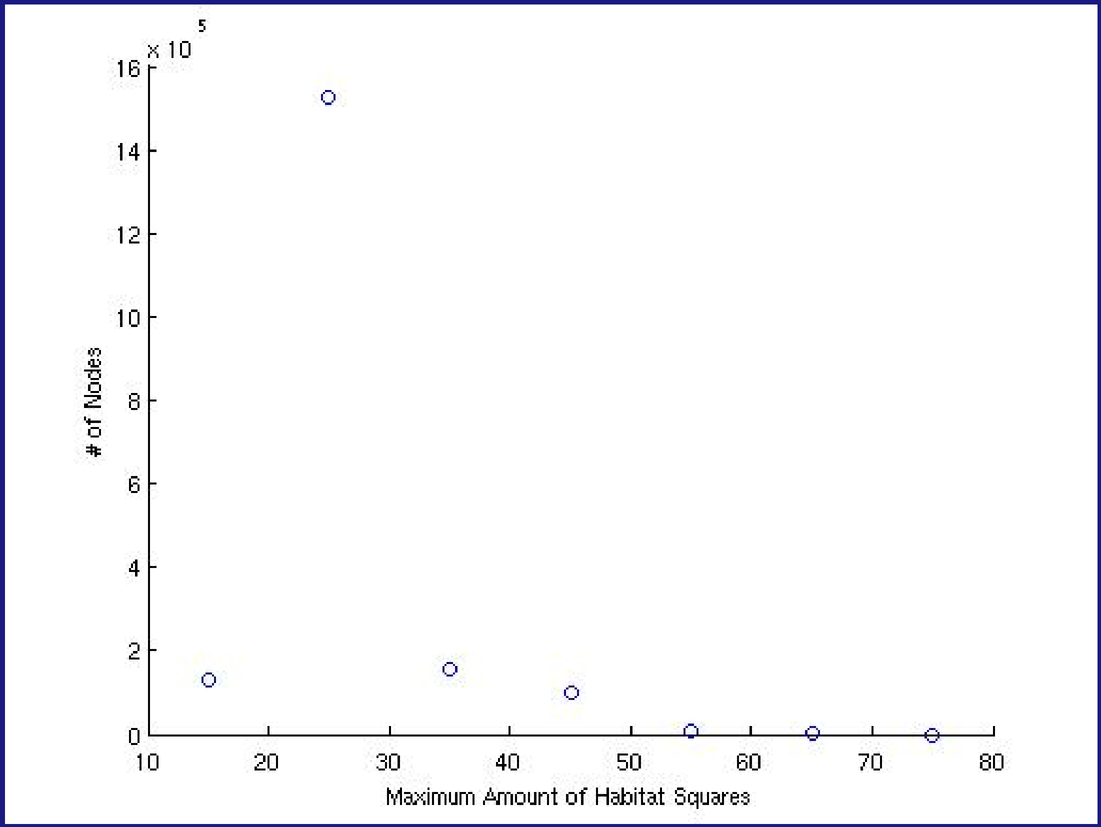


Figure 4: # BB nodes for solving the linearized model for varying maximum # candidate cells allowed to be green, with $n = 80$.

Current and Future Work

- Graded habitats – $0 \leq H_{ij} \leq 1$, with higher value indicating a more suitable cell

- Maintain HCI value (above M), while minimizing the number of candidate cells that have to be kept green:

$$\begin{array}{rrcl} \min & x_1 + x_2 + \ldots + x_n & & \\ \text{s.t.} & HCI(x_1, x_2, \ldots, x_n) & \geq & M \\ & x_j \in \{0, 1\} & & \text{for } j = 1, \ldots, n \end{array}$$

- Model competing species, e.g., the NSO and the barred owl