Integer Optimization (Spring 2009) — Homework 5

- For problems marked with **[A]**, you need to use AMPL and CPLEX to solve the problem. You must send me (by email to bkrishna@math.wsu.edu) your model and data files. The name of your files must indicate who you are – so, if you are Eric Cartman, you could name the model file for problem 1 as Prob1_Eric_Cartman.mod and so on.
- The total points (given in parentheses) add up to 140. You will be graded for 130 points.
- **This homework is due in class on Thursday, March 12**.

1. (40) **[A]** Write an AMPL model to solve the Sudoku puzzle. Your model should be generic, i.e., it should work for any *standard* Sudoku puzzle of size $n^2 \times n^2$ for any n. Go to http://www.websudoku.com, download and solve one 9×9 puzzle in each of the four levels – Easy, Medium, Hard, and Evil. Record the running times and the number of branch-and-bound nodes taken to solve each problem. You should submit (by email) one model file, and four data files. In each data file, note the *number* (or ID) of the corresponding puzzle.

2. (25) Prove that a matrix A with each $A_{ij} \in \{0, 1\}$ and each column having consecutive ones is totally unimodular (TU). (So, all the non-zero elements in each column, which are all 1's, appear in consecutive rows.)

3. (25) Show that the node-edge incidence matrix of an *undirected* graph is TU if and only if the graph is bipartite. (There is one column for each edge (i, j), with a $+1$ in both rows i and j.)

4. (25) Let $A \in \mathbb{Z}^{m \times n}$, $\boldsymbol{b} \in \mathbb{Z}^m$, $\boldsymbol{c} \in \mathbb{Z}^m$, and $d \in \mathbb{Z}$. Show that if the system $\{A\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{c}^T\boldsymbol{x} \leq d\}$ is totally dual integral (TDI), then the system $\{A\boldsymbol{x} \leq \boldsymbol{b}, \boldsymbol{c}^T\boldsymbol{x} = d\}$ is also TDI.

5. (25) Solve the following integer knapsack problem by branch-and-bound.

$$\begin{array}{lrcl} \max\ z = & 10x_1 + 12x_2 + 7x_3 + \frac{3}{2}x_4 & & \\ \text{s.t.} & 4x_1 + 5x_2 + 3x_3 + x_4 & \leq & 10, \\ & x_1, x_2 \in \mathbb{Z}_{\geq 0},\ x_3, x_4 \in \{0, 1\}. & & \end{array}$$

   You can use AMPL to solve the LP relaxations at each node. Show the entire branch-and-bound tree, along with the lower and upper bounds at each node. Indicate how you prune each node that is pruned, i.e., by bound, optimality, or infeasibility. **There is no need to send me the AMPL file for this problem.**