Integer Optimization (Spring 2009) — Homework 2

- The total points (given in parentheses) add up to 130. You will be graded for 120 points.
- **This homework is due in class on Thursday, January 29**.

1. (20) Consider the uncapacitated lot sizing (ULS) problem discussed in class. Explain how to model the following modifications to the original problem.
   (a) We allow demand to be *lost*, or unmet, in every period except the last one (period n), at a cost of b_t per unit of demand lost in period t.
   (b) Production can occur in at most T periods, but no two such periods can be consecutive.

2. (25) Give a formulation for the **interactive fixed charge function** using appropriate 0–1 variables (in addition to the original variables x_1, x_2). Similar to the basic fixed charge problem discussed in class, we now have $f(x_1, x_2)$ with

$$f(x_1,x_2) = \begin{cases} 0\ ,\ if\ x_1 = 0, x_2 = 0, \\ f_1\ ,\ if\ x_1 > 0, x_2 = 0, \\ f_2\ ,\ if\ x_1 = 0, x_2 > 0, \text{and} \\ f_{12},\ if\ x_1 > 0, x_2 > 0. \end{cases}$$

   Here, f_1, f_2, f_{12} are positive scalars and we assume

$$0 \le x_1 \le M_1, \quad 0 \le x_2 \le M_2, \text{ and} \tag{2.1}$$

$$f_1 \le f_{12}, \quad f_2 \le f_{12}. \tag{2.2}$$

   Both $f_{12} > f_1 + f_2$ and $f_{12} < f_1 + f_2$ can happen. Show that condition (2.2) is necessary for the formulation to be correct.

3. (25) Consider a piecewise linear (PL) function $f(x)$ defined on $[v_0, v_3]$ such that

$$f(v_0) = f_0,\ f(v_1) = f_{1,\ell},\ f(v_2) = f_2, \text{ and } f(v_3) = f_3,$$

   as discussed in class, but there is a jump at v_1. So, $f(x)$ is linear with slope

$$s_2 = \frac{f_2 - f_{1,r}}{v_2 - v_1}$$

   for all $v_1 < x \le v_2$, where $f_{1,r} > f_{1,\ell}$. Give an MIP representation of $f(x)$, assuming it appears in a minimization objective. Prove that your formulation will not work if it appeared in a maximization objective.

4. (30) Model the following statements using their CNFs.
   (a) $(L_1 \wedge L_2 \wedge (L_3 \vee L_4)) \vee (L_5 \wedge L_6)$.
   (b) $L_1 \vee \cdots \vee L_m \Leftrightarrow J_1 \wedge \cdots \wedge J_n$, where L_i means $x_i = 1$, J_k means $y_k = 1$, for 0–1 variables x_i and y_k.

5. (30) When modeling arbitrary disjunctions, prove that if **Assumption 2** (all the polyhedra have the same recession cone) is satisfied, then so is **Assumption 1** (there exist upper bound vectors $\boldsymbol{u}^i$). What about the other way around – does **Assumption 1** imply **Assumption 2**?