# Introduction to Linear Algebra (Math 220, Section 2) – Fall 2013
# Practice Midterm Examination

**Name:**

**WSU ID:**

- There are **eight** problems and **six** pages in this exam.
- Show all work.
- Provide appropriate **justifications** where required.
- Good luck!

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
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1. **(12)** Consider the following system of linear equations.

$$\begin{array}{rcrcrcr} 3x_1 & + & 4x_2 & + & 0.3x_3 & = & -3 \\ & & x_2 & + & 6x_3 & = & 5 \\ -2x_1 & - & 5x_2 & + & 7x_3 & = & 0 \end{array}$$

   (a) Write the system as a matrix equation.

   (b) Write the system as a vector equation.

   (c) Write the augmented matrix for the system.

2. **(16)** Let $A = \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 1 \\ -1 & -1 & 3 & 0 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 6 \\ 5 \\ -3 \end{bmatrix}$.

   (a) Solve the system $A\mathbf{x} = \mathbf{b}$, and write the solution in parametric vector form.

   (b) Using the result from Part (a), write the solution to the homogeneous system $A\mathbf{x} = \mathbf{0}$ in the parametric vector form.

3. **(10)** Let

$$\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}, \ \text{and } \mathbf{w} = \begin{bmatrix} 0.5 \\ 2 \\ -5 \end{bmatrix}.$$

   It can shown that $3\mathbf{u} - \mathbf{v} = 2\mathbf{w}$. Use this fact (and *no row operations*) to find a non-trivial solution to the homogeneous system $A\mathbf{x} = \mathbf{0}$, where

$$A = \begin{bmatrix} 2 & 1 & 0.5 \\ 5 & 3 & 2 \\ 7 & -1 & -5 \end{bmatrix}.$$

4. **(12)** Construct a $3 \times 3$ matrix $A$ with every entry non-zero such that the following vector $\mathbf{b}$ is ***not*** in the span of the columns of $A$. Justify your answer.

$$\mathbf{b} = \begin{bmatrix} 8 \\ -3 \\ 1 \end{bmatrix}$$

5. **(12)** Let

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}, \ \text{and} \ \mathbf{v}_4 = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}.$$

   (a) Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ span $\mathbb{R}^3$? Why or why not?

   (b) Does $\{\mathbf{v}_1, \mathbf{v}_2\}$ span $\mathbb{R}^3$? Why or why not?

6. **(11)** Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first does a horizontal shear transformation mapping $\mathbf{e}_2$ to $\mathbf{e}_2 + 2\mathbf{e}_1$ (leaving $\mathbf{e}_1$ unchanged), and then reflects points through the vertical axis.

7. **(15)** Consider the following system.

$$\begin{aligned} x_1 + 3x_2 &= k \\ x_1 - hx_2 &= 2 \end{aligned}$$

   Determine all the values of the parameters $h$ and $k$ for which each of the following statements are true.

   (a) The system has no solution.

   (b) The system has a unique solution.

   (c) The system has many solutions.

8. **(12)** Decide whether each of the following statements is *True* or *False*. Justify your answer.

   (a) A $3 \times 3$ matrix can have more than three echelon forms.

   (b) Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be two vectors in $\mathbb{R}^2$ that are not collinear (i.e., they do not lie along the same line), and let $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}$. Then the system $A\mathbf{x} = \mathbf{b}$ cannot have infinitely many solutions for any $\mathbf{b}$.

   (c) If a linear transformation is onto, then it cannot be one-to-one.

   (d) If $A$ is an $m \times n$ matrix, the range of the matrix transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$.