Introduction to Linear Algebra (Math 220, Section 2) – Fall 2013
Midterm Examination

Name:

WSU ID:

- There are **eight** problems and **six** pages in this exam.
- Show all work.
- Provide appropriate **justifications** where required.
- Good luck!

1	2	3	4	5	6	7	8	Total

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1. **(14)** Consider the following system of linear equations.

$$\begin{array}{rcrcrcl} & & 5x_2 & - & x_3 & = & 7 \\ 2x_1 & & & + & 9x_3 & = & 0 \\ x_1 & - & 4x_2 & + & 0.5x_3 & = & 3 \end{array}$$

   (a) Write the system as a matrix equation.
   (b) Write the system as a vector equation.
   (c) Write the augmented matrix for the system.

2. **(14)** Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$.

   (a) Solve the system $A\mathbf{x} = \mathbf{b}$, and write the solution in parametric vector form.
   (b) Using the result from Part (a), write the solution to the homogeneous system $A\mathbf{x} = \mathbf{0}$ in the parametric vector form.

3. **(10)** Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}, \ \text{and } \mathbf{w} = \begin{bmatrix} -4 \\ 17 \\ -13 \end{bmatrix}.$$

   It can shown that $4\mathbf{u} - 3\mathbf{v} - \mathbf{w} = \mathbf{0}$. Use this fact (and *no row operations*) to find a solution to the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 2 & -4 \\ 5 & 17 \\ -1 & -13 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ \text{and } \mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}.$$

4. **(12)** Construct a 3×3 matrix A with every entry non-zero such that the following vector $\mathbf{b}$ is in the span of the columns of A. Justify your answer.

$$\mathbf{b} = \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$$

5. **(12)** Let

$$\mathbf{v}_1 = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_4 = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}.$$

   (a) Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ span $\mathbb{R}^3$? Why or why not?

   (b) Does $\{\mathbf{v}_1, \mathbf{v}_2\}$ span $\mathbb{R}^3$? Why or why not?

6. **(10)** Is the transformation defined by $T(x_1, x_2) = (2x_1, 3x_1 - 5x_2, -x_2 + 1)$ linear? Explain.

7. **(16)** The images of the unit vectors in $\mathbb{R}^2$ under the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ are given as

$$T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 1 \\ h \end{bmatrix}, \quad \text{and } T(\mathbf{e}_2) = \begin{bmatrix} 3 \\ k \\ 0 \end{bmatrix}.$$

   Determine all the values of the parameters h and k for which each of the following statements are true.

   (a) T is one-to-one.

   (b) T maps $\mathbb{R}^2$ onto $\mathbb{R}^3$.

8. **(12)** Decide whether each of the following statements is *True* or *False*. Justify your answer.

   (a) Let A be an $n \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in $\mathbb{R}^n$, then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

   (b) The solutions of $A\mathbf{x} = \mathbf{b}$ are obtained by adding the vector $\mathbf{b}$ to the solutions of $A\mathbf{x} = \mathbf{0}$.

   (c) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

   (d) If a linear transformation is one-to-one, then it must also be onto.