# Introduction to Linear Algebra (Math 220_2) – Fall 2013
# Final Examination

**Name:** ______________________________

**WSU ID:** ______________________________

- There are **ten** problems in **six** pages.
- Show all work, and provide appropriate **justifications** where required.
- The use of electronic devices is **not** permitted.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
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1. **(10)**

$$\text{Let } A = \begin{bmatrix} 2 & 0 & -4 & 2 & -1 & -4 \\ 1 & 0 & -2 & 1 & 2 & 1 \\ 3 & 1 & -4 & 1 & 1 & 3 \\ -2 & 0 & 4 & -2 & -3 & -1 \\ 1 & 0 & -2 & 1 & 1 & 2 \end{bmatrix}. \text{ Then } A \text{ row reduces to } \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   (a) Find a basis for Col $A$.

   (b) Find a basis for Nul $A$.

   (c) What is dim Nul $A$? Explain.

   (d) What is rank $A$? Explain.

2. **(10)** Find all values of $h$ so that the set of vectors $\left\{ \begin{bmatrix} 4 \\ 4 \\ h \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ h \\ 6 \end{bmatrix} \right\}$ forms a basis for $\mathbb{R}^3$. Justify your answer.

3. **(10)**

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 0 & -2 & 2 \\ 1 & 3 & 4 & 0 \end{bmatrix}. \text{ Find } \det(A).$$

4. **(10)**

$$\text{Let } A = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 4 & 0 \\ 2 & 1 & 4 \end{bmatrix}.$$

   (a) Find the characteristic polynomial of $A$. You may leave your answer in factored form.

   (b) Find the eigenvalues of $A$. NOTE: The eigenvalues are integers between zero and ten.

5. **(10)**

$$\text{Let } B = \begin{bmatrix} 2 & 2 & 0 & 2 \\ 0 & 2 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 3 \end{bmatrix}.$$

   Find a basis for the eigenspace of $B$ associated with the eigenvalue $\lambda = 2$.

6. **(20)** Answer each of the following questions with justification.

   (a) If $A$ is a $5 \times 6$ matrix, can the columns of $A$ form a basis for $\mathbb{R}^5$?

   (b) If $A, B$, and $C$ are $n \times n$ matrices, $A$ is invertible, and $AB = AC$, then must $B = C$?

   (c) If $\mathbf{x}$ is an eigenvector of the $4 \times 4$ matrix $A$ corresponding to the eigenvalue $\lambda = 0$, do the columns of $A$ span $\mathbb{R}^4$?

   (d) If $A$ is a $3 \times 4$ matrix, what is the largest value of the rank of $A$? What is the largest value of the dimension of the null space of $A$?

   (e) Let $A$ be a $4 \times 4$ matrix with $\det(A) = 6$, and the matrix $B$ is formed from $A$ by first interchanging Rows two and three, and then dividing Row one by 2. What is $\det(B)$?

7. **(5)** Construct a $3 \times 3$ triangular matrix $A$ so that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in $\text{Col}(A)$.

8. **(5)** Let $A = \begin{bmatrix} 3 & 2 & -3 \\ 2 & 0 & 0 \\ 5 & -2 & -1 \end{bmatrix}$. Is $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ an eigenvector of $A$? Justify your answer.

9. **(5)** Let $A$ and $B$ be $3 \times 3$ matrices such that $\det(A) = 2$ and $\det(B) = -3$. Find each of the following determinants, or indicate that the determinant cannot be found from the information given.

   (a) $\det(B^3)$

   (b) $\det(3B)$

   (c) $\det(B^{-1}AB)$

   (d) $\det(A + B)$

   (e) $\det(A^{-2})$

10. **(5)** Let $\lambda$ be an eigenvalue of the $n \times n$ matrix $A$. Let $B = A - \lambda I$. Show that $B$ is not an invertible matrix.