FINAL EXAM - MATH 220-01

December 14, 2012 Dr. McDonald

Name:__________________________ ID:__________________________

I recognize that the use of electronic devices for anything other than playing music is not allowed during this exam. I also understand that copying off another student or using cheat sheets is considered cheating. I recognize that, above all else, engaging these types behaviours reflects badly on my own character and integrity.

Signature:__________________________

SHOW ALL WORK. CREDIT WILL NOT BE GIVEN FOR UNSUBSTANTIATED ANSWERS

(10 pts) 1. Let

$$A = \begin{bmatrix} 2 & 4 & -2 & 4 & 2 & -6 \\ -4 & -8 & -3 & 6 & 3 & -2 \\ 6 & 12 & 1 & -2 & 3 & 4 \end{bmatrix}. \text{ The } rref(A) = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}.$$

Write the solutions to the following system of linear equations in parametric vector form:

$$\begin{array}{rcrcrcrcrcr} 2x_1 & + & 4x_2 & - & 2x_3 & + & 4x_4 & + & 2x_5 & = & -6 \\ -4x_1 & - & 8x_2 & - & 3x_3 & + & 6x_4 & + & 3x_5 & = & -2 \\ 6x_1 & + & 12x_2 & + & x_3 & - & 2x_4 & + & 3x_5 & = & 4 \end{array}$$

(10 pts) 2. Let

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 5 \\ 1 & 3 & 4 & 2 \end{bmatrix}$$

Find $\det(A)$.

(10 pts) 3. Let

$$A = \begin{bmatrix} 2 & 3 & -5 & 6 & -5 \\ 2 & 4 & -6 & 0 & 4 \\ 4 & 7 & -11 & 6 & 3 \\ 0 & 1 & -1 & -6 & 9 \\ 4 & 8 & -12 & 0 & 12 \end{bmatrix}, \operatorname{rref}(A) = \begin{bmatrix} 1 & 0 & -1 & 12 & 0 \\ 0 & 1 & -1 & -6 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for $\mathrm{Col}(A)$

(b) Find a basis for $\mathrm{Nul}(A)$.

(c) What is the rank of A

(d) What is the dimension of the null space of A.

(15 pts) 4. Let

$$A = \begin{bmatrix} 2 & 2 & -4 & -2 \\ 0 & 3 & 0 & 0 \\ 0 & -1 & 5 & 1 \\ 0 & -1 & 1 & 5 \end{bmatrix}.$$

(a) Find the characteristic polynomial of A.

(b) Find the eigenvalues of A. NOTE: The eigenvalues are integers between 0 and 10.

(c) Let

$$B = \begin{bmatrix} 3 & 1 & 0 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & -1 & -1 & 1 \end{bmatrix}.$$

Find a basis for the eigenspace of B associated with the eigenvalue $\lambda = 3$

(15 pts) 5. Justify your answer for each of the following.

(a) If A has more columns than rows, can the columns of A be linearly independent?

(b) If A is a 5×5 matrix and the rank of A is 5, is $\det(A) = 0$?

(c) Do six linearly independent vectors in $\mathbb{R}^9$ span a subspace of dimension six?

(d) If A, B, and C are $n \times n$ matrices and $AB = AC$, must $B = C$?

(e) Let u, v and w be vectors in $\mathbb{R}^3$. If u is orthogonal to v and v is orthogonal to w, must u be orthogonal to w?

(5 pts) 6. Let A and B be 5×5 matrices such that $\det(A) = 3$ and $\det(B) = 2$. Find each of the following determinants, or indicate that it cannot be found from the information given.

(a) $\det(A^3)$

(b) $\det(2A)$

(c) $\det(BA)$

(d) $\det(A + B)$

(e) $\det(B^{-1})$

(5 pts) 7. Suppose that A is matrix with $\text{rank}(A) = 3$, $\dim \text{Nul}(A) = 2$, and such that the row reduced echelon form of A has one row of zeros. How many rows does A have? How many columns does A have?

(5 pts) 8. Let A and B be $n \times n$ matrices such that B is invertible. Prove that $\det(B^{-1}AB) = \det(A)$.

(5 pts) 9. Let $x \in \mathbb{R}^n$ be an eigenvector of both the $n \times n$ matrices A and B. Show that x is an eigenvector of the matrix AB.