1. Solve for the variable in each equation below: (5 points each)

\( \frac{2t-1}{x} = \frac{6t-3}{5} + \frac{1}{10} \)

\[
5(2t-1) = 2(6t-3) + 5t \\
10t - 5 = 12t - 6 + 5t \\
6 - 5 = 3t \\
t = \frac{1}{3}
\]

b.) \( \frac{8}{x^2 - 1} = \frac{1}{x+1} - \frac{1}{x-1} \)

\[
8 = (x-1) - (x+1) = -2 \\
\text{No solution!}
\]

2. In the literal equation below, solve for \( t \) in terms of the remaining variables. (5 points) (Circle the correct answer.)

\( mt + gt = 2(3 + t) \)

A) \( 6 - m - g \)  
B) \( \frac{6}{mg - 2} \)  
C) \( \frac{6}{mg + 2} \)  
D) \( \frac{6}{m + g + 2} \)  
E) \( \frac{6}{m + g - 2} \)

\[
t(m + g) = 6 + 2t \\
t(m + g - 2) = 6 \\
t = \frac{6}{m + g - 2}
\]

3. The sum of the root(s), that is the solution(s), of the equation \( \sqrt{x^2 - 4} = \left( x - 4 \right) ^2 \) is: (5 points) (Circle the correct answer.)

A) 0  
B) 9  
C) -5  
D) 5  
E) 7

\[
\sqrt{x + 2} = x^2 - 8x + 16 \\
0 = x^2 - 8x - x + 16 - 2 \\
0 = x^2 - 9x + 14 \\
(x - 7)(x - 2) = 0 \\
x = 7, 2 \\
x = 7, 2
\]

\[
\text{But } x = 2 \text{ does not work!} \\
\sqrt{2 + 2} = 2 - 4 = -2 \\
\sqrt{4} = -2 \\
2 \neq -2 \\
\text{So, only root is } x = 7
\]
4. Solve for $x$: $x^4 - 7x^2 + 12 = 0$ (5 points)

$$u = x^2 \quad \Rightarrow \quad u^2 - 7u + 12 = 0$$
$$\quad \Rightarrow (u-3)(u-4) = 0, \quad u = 3, 4$$
$$x^2 = 3, 4$$

So \quad $x = \pm \sqrt{3}, \pm \sqrt{4}$ \quad i.e., $x = \pm \sqrt{3}, \pm \sqrt{2}$

5. In each application problem below represent all unknowns in terms of one variable, establish an appropriate equation and solve. Show all appropriate steps. (7 points each)

a.) A homeowner needs to spray her trees to kill defoliating insects. She needs 128 ounces of a solution made up of 3 parts insecticide A and 5 parts of insecticide B. How many ounces of insecticide B should be used?

Let $x =$ # ounces of insecticide B

Then \quad # ounces of insecticide $A = \frac{(3)}{5}x$

$$\text{Total ounces} = x + \frac{(3)}{5}x = 128$$
$$\quad \Rightarrow 5x + 3x = 128 \times 5; \quad x = \frac{128 \times 5}{8} = 80$$

b.) A company's revenue from the sale of interior doors is $98 per door. The cost of producing the doors is $48 per door with $14,000 in fixed costs. How many doors must the company sell in order to break even?

Let $x =$ # doors

$$\text{Total cost} = \text{fixed cost} + \text{variable cost} = 14000 + 48x$$
$$\text{Total revenue} = 98x.$$ 

For Break even: $\text{revenue} = \text{cost}$

$$98x = 48x + 14000$$
$$50x = 14000$$
$$x = \frac{14000}{50} = 280$$

6. Solve for $x$ in each inequality below. State your solutions using interval notation. (5 points each)

a.) \quad \frac{6x - 2}{3} \geq -\frac{1}{4}

\quad \Rightarrow \quad 6x - 2 \geq -\frac{3}{4}

\quad \Rightarrow \quad 6x \geq \frac{2}{3} - \frac{3}{4} = \frac{5}{4}

\quad \Rightarrow \quad x \geq \frac{5}{4} \times \frac{1}{6} = \frac{5}{24}

\quad \Rightarrow \quad \left[ \frac{5}{24}, \infty \right)

b.) \quad \frac{2x + 8}{3} \geq 4

\quad \Rightarrow \quad 2x + 8 \geq 12

\quad \Rightarrow \quad 2x \geq 4

\quad \Rightarrow \quad x \geq 2

\quad \Rightarrow \quad (-\infty, -10] \cup [2, \infty)
7. Evaluate $\sum_{i=1}^{20} 14i$. 

\[
\sum_{i=1}^{20} 14i = 14 \sum_{i=1}^{20} i = 14 \left( \frac{20(20+1)}{2} \right) = 14 \cdot 210 = 2940
\]

8. State the degree and leading coefficient of the polynomial function $f(x) = -3x^8 + 2x^5 + 4$. (1 point each)

Leading Coefficient: $-3$

Degree: 8

9. Given the function $g(x) = \frac{x}{x - 3}$, find the domain of function $g$, $g(0)$, $g(1/2)$ and $g(x^2)$. Simplify your answer if needed. (2 points each)

\[
g(\frac{1}{2}) = \frac{\frac{1}{2}}{\frac{1}{2} - 3} = \frac{\frac{1}{2}}{\frac{1}{2} - \frac{3}{2}} = \frac{1}{1 - 6} = -\frac{1}{5}
\]

\[
g(x^2) = \frac{x^2}{x^2 - 3}
\]

Domain: All reals except 3

\[
g(0) = 0
\]

\[
g(1/2) = -\frac{1}{5}
\]

\[
g(x^2) = \frac{x^2}{x^2 - 3}
\]

10. If $f(x) = x^2 - 1$ and $g(x) = x + 2$, find: (3 points each)

a.) $(f + g)(x) = f(x) + g(x) = x^2 - 1 + x + 2 = x^2 + x + 1$

b.) $(f \times g)(x) = f(x) \times g(x) = (x^2 - 1) \cdot (x + 2) = (x + 1) \cdot (x + 2) = x^2 + 4x + 3$

c.) $(f \circ g)(x) = f(g(x)) = f(x + 2) = (x + 2)^2 - 1 = x^2 + 4x + 4 - 1$

d.) $(g \circ f)(x) = g(f(x)) = g(x^2) + 2 = x^2 - 1 + 2 = x^2 + 1$

11. Find the inverse of the function below if it exists, showing all appropriate steps. If the inverse doesn’t exist, so state. (5 points)

Let $g(x)$ be the inverse. Then

\[
f(x) = 3x - 5
\]

\[
f(g(x)) = x
\]

i.e., $3g(x) - 5 = x$

Solving for $g(x)$,

\[
3g(x) = x + 5
\]

so, $g(x) = \frac{1}{3} x + \frac{5}{3}$

\[
f^{-1}(x) = \frac{1}{3} x + \frac{5}{3}
\]
12. Given the equation: \( y = 4 - 2x^2 \): (8 points)

a.) Sketch the graph on the axes at right.
b.) Identify the intercepts
   \( y = 0: 4 - 2x^2 = 0 \)
   \( x \) intercept: (\( \sqrt{2}, 0 \), \( -\sqrt{2}, 0 \)) \( x^2 = 2, x = \pm \sqrt{2} \)
   \( y \) intercept: (0, 4) \( x = 0: y = 4 \)
c.) Based on your graph, is \( y \) a function of \( x \)? \( \text{Yes. Vertical lines intersect graph at one point each.} \) 
   \( \text{No. Horizontal lines intersect graph at more than one point.} \)
d.) If \( y \) is a function of \( x \), is it one-to-one?
   \( \text{No.} \)
e.) If \( y \) is a function of \( x \), what are the domain and range?
   - Domain: All real values
   - Range: All real values \( \leq 4 \) \( \quad (-\infty, 4] \)

13. Graph the case defined function below on the axes provided. (4 points)

\[ y = f(x) = \begin{cases} 
  x + 1 & \text{if } 0 \leq x < 7 \\
  5 & \text{if } x \geq 7 
\end{cases} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

14. Complete the table below indicating the types of symmetry of the graph of each equation. The first row is completed to illustrate an appropriate response. (1 point each cell)

<table>
<thead>
<tr>
<th>Equation</th>
<th>( x )-axis symmetry</th>
<th>( y )-axis symmetry</th>
<th>Origin symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 5x )</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>( y = x^2 - 4 )</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( y = (x-4)^2 = x^2 - 4 )</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( x^2 + xy + y^2 = 0 )</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>( y = \sqrt{x^2 - 25} )</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( -y = \sqrt{x^2 - 25} )</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

\[ y = \pm \sqrt{x^2 - 25} \]