

Exam 4

Construction and Evaluation of Actuarial Models

Exam 4 is a three-and-a-half-hour, multiple-choice exam administered by Preliminary Actuarial Examinations/SOA and is identical to SOA Exam C. The examination is jointly sponsored and administered by the CAS, SOA, and the Canadian Institute of Actuaries (CIA). The examination is also jointly sponsored by the American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA).

Exam 4 is administered as a computer-based test. For additional details, please refer to “Computer-Based Testing Rules and Procedures” (<http://www.beanactuary.org/exams/cbt.cfm>).

The syllabus for this examination provides an introduction to modeling and covers important actuarial methods that are useful in modeling. A thorough knowledge of calculus, probability, and mathematical statistics is assumed.

The candidate will be introduced to useful frequency and severity models beyond those covered in Exam 3. The candidate will be required to understand the steps involved in the modeling process and how to carry out these steps in solving business problems. The candidate should be able to: 1) analyze data from an application in a business context; 2) determine a suitable model including parameter values; and 3) provide measures of confidence for decisions based upon the model. The candidate will be introduced to a variety of tools for the calibration and evaluation of the models.

A variety of tables is available for the candidate on the CAS and SOA Web Sites and will be provided to the candidate at the examination. These include values for the standard normal distribution, chi-square distribution, and abridged inventories of discrete and continuous probability distributions. Since they will be provided at the examination, candidates will not be allowed to bring copies of the tables into the examination room.

Please check the “Syllabus Updates” section of the CAS Web Site for any changes to the *Syllabus*. Information about Study Notes is available in the “Study Resources” section.

LEARNING OBJECTIVES

The candidate is expected to be familiar with survival, severity, frequency and aggregate models, and use statistical methods to estimate parameters of such models given sample data. The candidate is further expected to identify steps in the modeling process, understand the underlying assumptions implicit in each family of models, recognize which assumptions are applicable in a given business application, and appropriately adjust the models for impact of insurance coverage modifications.

Specifically, the candidate is expected to be able to perform the tasks listed below:

A. Severity Models

1. Calculate the basic distributional quantities:
 - a. Moments
 - b. Percentiles
 - c. Generating functions
2. Describe how changes in parameters affect the distribution.
3. Recognize classes of distributions and their relationships.
4. Apply the following techniques for creating new families of distributions:
 - a. Multiplication by a constant
 - b. Raising to a power
 - c. Exponentiation
 - d. Mixing

5. Identify the applications in which each distribution is used and reasons why.
6. Apply the distribution to an application, given the parameters.
7. Calculate various measures of tail weight and relative tail weight, and interpret the results to compare the tail weights.

B. Frequency Models

For the Poisson, Mixed Poisson, Binomial, Negative Binomial, Geometric distribution and mixtures thereof:

1. Describe how changes in parameters affect the distribution,
2. Calculate moments,
3. Identify the applications for which each distribution is used and reasons why,
4. Apply the distribution to an application given the parameters,
5. Apply the zero-truncated or zero-modified distribution to an application given the parameters.

C. Aggregate Models

1. Compute relevant probabilities and moments for aggregate risk models.
2. Evaluate aggregate claim models.
3. Compute aggregate claims distributions.

D. Severity, Frequency, and Aggregate Models

1. Evaluate the impacts of coverage modifications:
 - a. Deductibles
 - b. Limits
 - c. Coinsurance
2. Calculate Loss Elimination Ratios.
3. Evaluate effects of inflation on losses.

E. Risk Measures

1. Calculate VaR and TVaR and explain their use and limitations.

F. Construction of Empirical Models

1. Estimate failure time and loss distributions using:
 - a. Kaplan-Meier estimator, including approximations for large data sets
 - b. Nelson-Åalen estimator
 - c. Kernel density estimators
2. Estimate the variance of estimators and confidence intervals for failure time and loss distributions.
3. Apply the following concepts in estimating failure time and loss distribution:
 - a. Unbiasedness
 - b. Consistency
 - c. Mean squared error

G. Construction and Selection of Parametric Models

1. Estimate the parameters of failure time and loss distributions using:
 - a. Maximum likelihood
 - b. Method of moments
 - c. Percentile matching
 - d. Bayesian procedures
2. Estimate the parameters of failure time and loss distributions with censored and/or truncated data using maximum likelihood.

3. Estimate the variance of estimators and the confidence intervals for the parameters and functions of parameters of failure time and loss distributions.
4. Apply the following concepts in estimating failure time and loss distributions:
 - a. Unbiasedness
 - b. Asymptotic unbiasedness
 - c. Consistency
 - d. Mean squared error
 - e. Uniform minimum variance unbiased
5. Determine the acceptability of a fitted model and/or compare models using:
 - a. Graphical procedures
 - b. Kolmogorov-Smirnov test
 - c. Anderson-Darling test
 - d. Chi-square goodness-of-fit test
 - e. Likelihood ratio test
 - f. Schwarz Bayesian Criterion

H. Credibility

1. Apply limited fluctuation (classical) credibility including criteria for both full and partial credibility.
2. Perform Bayesian analysis using both discrete and continuous models.
3. Apply Bühlmann and Bühlmann-Straub models and understand the relationship of these to the Bayesian model.
4. Apply conjugate priors in Bayesian analysis and in particular the Poisson-gamma model.
5. Apply empirical Bayesian methods in the nonparametric and semiparametric cases.

I. Simulation

1. Simulate both discrete and continuous random variables using the inversion method.
2. Estimate the number of simulations needed to obtain an estimate with a given error and a given degree of confidence.
3. Use simulation to determine the p-value for a hypothesis test.
4. Use the bootstrap method to estimate the mean squared error of an estimator.
5. Apply simulation methods within the context of actuarial models.

Text References for Exam 4/C

For Learning Objectives A through G and I, the readings are:

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions* (Third Edition), 2008, John Wiley and Sons:

Chapter 3,
 Chapter 4,
 Chapter 5, Sections 5.1– 5.4,
 Chapter 6, Sections 6.1– 6.5 and 6.7,
 Chapter 8,
 Chapter 9, Sections 9.1–9.7 (excluding 9.6.1 and examples 9.9 and 9.11), Sections 9.11.1–
 9.11.2,
 Chapter 12,
 Chapter 13,
 Chapter 14,
 Chapter 15, Sections 15.1–15.6.4, and 15.6.6,
 Chapter 16,
 Chapter 21, Sections 21.1–21.2 (excluding 21.2.4).

For Learning Objective H (Credibility), the candidate may use any of the options shown below:

Option A	
Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., <i>Loss Models: From Data to Decisions</i> (Third Edition), 2008, John Wiley and Sons, New York, Chapter 20 (Sections 20.2; 20.3 excluding 20.3.8; and 20.4 excluding 20.4.3).	
Option B	
Mahler, H.C.; and Dean, C.G., “Credibility,” <i>Foundations of Casualty Actuarial Science</i> (Fourth Edition), 2001, Casualty Actuarial Society, Chapter 8, Section 1 (background only) and Sections 2-5. (Also available as an SOA Study Note.)	W
Dean, C.G., “Topics in Credibility Theory,” 2004 (SOA Study Note).	W
Option C	
Herzog, T.N., <i>Introduction to Credibility Theory</i> (Third Edition), 1999, Chapters 1-3 (background only); 4-8; and 9 (background only).	

Additional Study Notes

Corrections and comments for <i>Loss Models: From Data to Decisions</i> , Third Edition (see http://www.soa.org/files/pdf/edu-loss-models-errata-corrections.pdf).	W
Tables for Exam C/4—also see the HTML format of the tables that will be used for the exam by computer-based testing.	W
Sample questions and solutions.	W
All released exams since 2000 may be found on the SOA Web Site and in “Past Exams” under “Study Tools” in the “Admissions/Exams” section of the CAS Web Site.	W

Source Key

NEW Indicates new or updated material or modified citation.

W Represents material that is available at no charge under “Web Notes” in the “Study Tools” section of the CAS Web Site.

Publishers and Distributors

Contact information is furnished for those who wish to purchase the text references cited for Exam 4/C. Publishers and distributors are independent and listed for the convenience of candidates; inclusion does not constitute endorsement by the CAS.

ACTEX Publications (Mad River Books), 107 Groppo Drive, Suite A, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; e-mail: retail@actexamdriver.com; Web site: www.actexamdriver.com.

Actuarial Bookstore, P.O. Box 69, Greenland, NH 03840; telephone: (800) 582-9672 (U.S. only) or (603) 430-1252; fax: (603) 430-1258; Web site: www.actuarialbookstore.com.

Casualty Actuarial Society *Foundations of Casualty Actuarial Science* (Fourth Edition), 4350 N. Fairfax Drive, Suite 250, Arlington, VA 22203; telephone: (703) 276-3100; fax: (703) 276-3108; e-mail: office@casact.org; Web site: www.casact.org.

Herzog, T.N., *Introduction to Credibility Theory* (Third Edition), 1999, ACTEX Publications, 140 Willow Street, Suite One, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; e-mail: retail@actexamdriver.com.

Klugman, S.A.; Panjer, H.H.; and Willmot, G.E., *Loss Models: From Data to Decisions* (Third Edition), 2008, John Wiley and Sons, One Wiley Drive, Somerset, NJ 08875; telephone: (800) 225-5945 or (732) 469-4400.

SlideRule Books, P.O. Box 69, Greenland, NH 03840; telephone: (877) 407-5433 or (603) 373-6140; fax: (877) 417-5433 or (603) 430-1258; Web site: www.sliderulebooks.com.