Exam 3L
Actuarial Models—Life Contingencies and Statistics Segment

Exam 3L is a two-and-a-half-hour, multiple-choice exam on life contingencies and statistics that is administered by the CAS. This material develops the candidate's knowledge of the theoretical basis of certain actuarial models and the application of those models to insurance and other risks. A thorough knowledge of calculus, probability and interest theory is assumed. Knowledge of risk management at the level of Exam 1/P is also assumed.

Before commencing study for this exam, candidates should read the "Introduction" to "Materials for Study" for important information about learning objectives, knowledge statements, readings, and the range of weights. Items marked with a bold **W**—the 2010 CAS Exam 3L Web Notes—are available at no charge in the "Study Tools" section of the CAS Web Site or may be purchased in printed form from the CAS Office. Pricing and order information is available in both the "Study Resources" and "Exam Applications and Order Forms" sections.

Please check the "*Syllabus* Updates" section of the CAS Web Site for any changes to the *Syllabus*.

The CAS will grant credit for CAS Exam 3L to those who successfully complete SOA Exam MLC (life contingencies segment) in the current education structure.

A thorough knowledge of calculus, probability, and interest theory is assumed. Knowledge of risk management at the level of Exam 1/P is also assumed.

This examination develops the candidate's knowledge of the theoretical basis of contingent payment models and the application of those models to insurance risks.

The candidate will be required to develop an understanding of contingent payment models. The candidate will be expected to understand what important results can be obtained from these models for the purpose of making business decisions, and what approaches can be used to determine these results.

A variety of tables will be provided to the candidate with the exam. Copies of the specific tables are available on the CAS Web Site under "Web Notes." They include values for the standard normal distribution, illustrative life tables, abridged inventories of discrete and continuous probability distributions, Chi-square Distribution, *t*-Distribution, and *F*-Distribution. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

The CAS will test the candidate's knowledge of topics that are presented in the learning objectives. Thus, the candidate should expect that each exam will cover a large proportion of the learning objectives and associated knowledge statements and syllabus readings, and that all of these will be tested at least once over the course of a few years—but each one may not be covered on each exam

A guessing adjustment will be used in grading Exam 3L. Details are provided under "Guessing Adjustment" in the "Rules-The Examination" section.

A. Survival Models

Range of weight for Section A: 33-37 percent

Candidates should be able to work with discrete and continuous univariate probability distributions for failure time random variables. They will be expected to set up and solve equations in terms of life table functions, cumulative distribution functions, survival functions, probability density functions, and hazard functions (e.g., force of mortality), as appropriate. They should have similar facility with models of the joint distribution of two failure times (multiple lives) and the joint distribution of competing risks (multiple decrement).

© 2009, Casualty Actuarial Society, All Rights Reserved

Candidates should be able to use Markov Chains in order to determine state probabilities and transition probabilities.

LEARNING OBJECTIVES	**KNOWLEDGE STATEMENTS**
1. For discrete and continuous univariate probability distributions for failure time random variables, develop expressions in terms of the life table functions, l_x, q_x, p_x, ${}_nq_x$, ${}_np_x$, and ${}_{m\|n}q_x$, for the cumulative distribution function, the survival function, the probability density function and the hazard function (force of mortality), and be able to: • Establish relations between the different functions • Develop expressions, including recursion relations, in terms of the functions for probabilities and moments associated with functions of failure time random variables, and calculate such quantities using simple failure time distributions • Express the effect of explanatory variables on a failure time distribution in terms of proportional hazards and accelerated failure time models The distributions may be left-truncated, right-censored, both, or neither. Range of weight: 5-10 percent	a. Failure time random variables b. Life table functions c. Cumulative distribution functions d. Survival functions e. Probability density functions f. Hazard functions g. Relationships between failure time random variables in the functions above
READINGS	
Option 1: Bowers et al., Chapter 3 (excluding 3.6 and 3.8) Option 2: Cunningham et al, Chapters 3.1-3.4, 4.1-4.4 (Candidates may find the two-page study note, “Notational Differences,” helpful in identifying notational differences used in these two books, but it is not required.)	

LEARNING OBJECTIVES	**KNOWLEDGE STATEMENTS**
2. Assuming a uniform distribution of deaths, define the continuous survival time random variable that arises from the discrete survival time random variable. Range of weight: 3-7 percent	a. Life table function forms under uniform distribution of deaths assumption
READINGS	
Option 1: Bowers et al., Chapter 3.6 Option 2: Cunningham et al., Chapter 4.5	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. Given the joint distribution of two failure times: • Calculate probabilities and moments associated with functions of these random variables' variances. • Characterize the distribution of the smaller failure time (the joint life status) and the larger failure time (the last survivor status) in terms of functions analogous to those in the Learning Objective 1 above, as appropriate. • Develop expressions, including recursion relations, for probabilities and moments of functions of the joint life status and the last survivor status, and express these in terms of the univariate functions in Learning Objective 1 above (assuming independence of the two failure times). Range of weight: 5-10 percent	a. Joint distribution of failure times b. Probabilities and moments

READINGS
Option 1: Bowers et al., Chapter 9.1-9.5 Option 2: Cunningham et al., Chapters 9.1-9.2, 9.5

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
4. Based on the joint distribution (pdf and cdf) of the time until failure and the cause of failure in the competing risk (multiple decrement) model and in terms of the functions $l_x^{(t)}$, ${}_tq_x^{(t)}$, ${}_tp_x^{(t)}$, ${}_td_x^{(t)}$, ${}_tm_x^{(t)}$ (t): • Establish relations between the functions. • Calculate probabilities and moments associated with functions of these random variables, given the joint distribution of the time of failure and the cause of failure. Range of weight: 5-10 percent	a. Time until failure b. Competing risk (multiple decrement) models

READINGS
Option 1: Bowers et al., Chapter 10.1-10.3 Option 2: Cunningham et al., Chapters 10.1-10.3

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
5. For homogenous and non-homogenous discrete-time Markov chain models: • Define each model. • Calculate probabilities of being in a particular state at a particular time. • Calculate probabilities of transitioning between states. Range of weight: 5-10 percent	a. Markov chains b. Transition probability matrix c. Discrete-time Markov chains
READINGS	
Daniel Markov, Chapters 1 and 3	

B. Stochastic Processes

Range of weight for Section B: 5-10 percent

Candidates should be able to solve problems using stochastic processes. They should be able to determine the probabilities and distributions associated with these processes. Specifically, candidates should be able to use a Poisson process in these applications.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Describe the properties of Poisson processes: • For increments in the homogeneous case • For interval times in the homogeneous case • For increments in the non-homogeneous case • Resulting from special types of events in the Poisson process • Resulting from sums of independent Poisson processes Range of weight: 0-5 percent	a. Poisson process b. Non-homogeneous Poisson process
2. For any Poisson process and the interarrival and waiting distributions associated with the Poisson process, calculate: • Expected values • Variances • Probabilities Range of weight: 0-5 percent	a. Probability calculations for Poisson process
READINGS	
Daniel Poisson	

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. For a compound Poisson process, calculate moments associated with the value of the process at a given time. Range of weight: 0-5 percent	a. Compound Poisson process
READINGS	
Daniel Poisson	

C. Life Contingency Models

Range of weight for Section C: 23-27 percent

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Apply a principle to a present value model to associate a cost or pattern of costs (possibly contingent) with a set of future contingent cash flows. Range of weight: 10-15 percent	a. Principles include: equivalence, exponential, standard deviation, variance, and percentile b. Models including those listed in Learning Objectives A2-A4 (survival models). c. Principle applications include: life insurance, annuities, health care, credit risk, environmental risk, consumer behavior (e.g., subscriptions), and warranties

READINGS
Option 1: Bowers et al., Chapters 4.1-4.3, 5.1-5.3, 6.1-6.3, 9.7 Option 2: Cunningham et al., Chapters 5.1-5.4, 6.1-6.3, 7.1-7.3, 9.4.1-9.4.4

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
2. Analyze present value of future loss random variables for life insurances and annuities and determine net liabilities using prospective and retrospective methods. Range of weight: 5-10 percent	a. Life insurance liability calculations b. Prospective and retrospective methods

READINGS
Option 1: Bowers et al., Chapter 7.1-7.4 Option 2: Cunningham et al., Chapter 8.1, 8.3

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
3. Using present-value-of-benefit random variables and present-value-of-future-loss random variables extended to discrete time Markov chains, calculate: • Actuarial present values of cash flows at transitions between states • Actuarial present values of cash flows while in a state • Considerations (premiums) using the Equivalence Principle • Liabilities (reserves) using the prospective method Range of weight: 3-7 percent	a. Cash flows at transition b. Triple product summation c. Transition probabilities

READINGS
Daniel Markov, Chapters 2 and 3

D. Statistics

Range of weight for Section D: 33-37 percent

Candidates should be able to apply statistical theory to solve business problems.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Perform point estimation of statistical parameters using the following statistical methods: • Maximum likelihood estimation ("MLE") • Method of moments Apply criteria to the estimates such as: • Consistency • Unbiasedness • Minimum variance • Mean square error Range of weight: 10-15 percent	a. Equations for MLE of mean, variance from a sample b. Estimation of mean and variance based on sample c. General equations for MLE of parameters d. Equations for estimation of parameters using method of moments for means, variances, and higher moments e. Recognition of consistency property of estimators and alternative measures of consistency f. Application of criteria for measurement when estimating parameters through minimization of variance, mean square error g. Definition of statistical bias and recognition of estimators that are unbiased or biased
2. Test statistical hypotheses including Type I and Type II errors using: • Neyman-Pearson lemma • Likelihood ratio tests Apply Neyman-Person lemma to construct likelihood ratio equation. Range of weight: 10-15 percent	a. Presentation of fundamental inequalities based on general assumptions and normal assumptions b. Definition of Type I and Type II errors c. Significance levels d. One-sided versus two-sided tests e. Estimation of sample sizes under normality to control for Type I and Type II errors f. Determination of critical regions g. Definition and measurement of likelihood ratio tests h. Determining parameters and testing using tabular values i. Recognizing when to apply likelihood ratio tests versus chi-square or other goodness of fit tests
3. Calculate order statistics of a sample and use critical values from a sampling distribution to test means and variances. Range of weight: 3-7 percent	a. General form for distribution of n^{th} largest element of a set b. Application to a given distributional form c. Recognition of random variables from sample that behave as t-stat or F-stat d. Determination of parameters when applying these tests and obtaining tabular values e. Presentation of hypotheses testing from above for mean and variances
4. Perform a linear regression using the least squares method. Range of weight: 3-7 percent	a. Presentation and calculation of equations for regression statistics

READINGS
There is no single required text for Section D. The texts listed below may be considered as representative of the many texts available to cover the material on which the candidate may be examined based on the learning objectives and knowledge statements: Hogg and Tanis Hogg et al. Larsen and Marx

Complete Text References for Exam 3L

Text references are alphabetized by the citation column.

Citation	Abbreviation	Learning Objectives	Source
Bowers, N.L.; Gerber, H.U.; Hickman, J.C.; Jones, D.A.; and Nesbitt, C.J., *Actuarial Mathematics* (Second Edition), 1997, Society of Actuaries, including erratum.	Bowers et al.	A1-A4, C1, C2	L
Cunningham, R.; Herzog, T.; and London, R., *Models for Quantifying Risk* (Third Edition), ACTEX Publications, 2008, with the following citation: Chapters 3.1-3.4, 4.1-4.4, 4.5, 5.1-5.4, 6.1-6.3, 7.1-7.3, 8.1, 8.3, 9.1-9.2, 9.4.1-9.4.4, 9.5, and 10.1-10.3. Candidates are not responsible for formulae 4.62 through 4.65 nor are they responsible for the "Hyperbolic (Balducci)" column of Table 4.3.	Cunningham et al.	A1-A4, C1, C2	L
Daniel, J.W., "Multi-state Transition Models with Actuarial Applications," Study Note, 2004 (second printing with minor corrections, October 2007).	Daniel Markov	A5, C3	W
Daniel, J.W., "Poisson processes (and mixture distributions)," Study Note, June 2008.	Daniel Poisson	B1-B3	W
Hogg, R.V.; McKean, J.W.; and Craig, A.T., *Introduction to Mathematical Statistics* (Sixth Edition), 2004, Prentice Hall.	Hogg et al.	D1-D4	
Hogg, R.V.; and Tanis, E., *Probability and Statistical Inference* (Eighth Edition), 2010, Prentice Hall.	Hogg and Tanis	D1-D4	
Larsen, R.J.; and Marx, M.L., *An Introduction to Mathematical Statistics and Its Applications* (Fourth Edition), 2006, Prentice Hall.	Larsen and Marx	D1-D4	
"Notational Differences Between *Actuarial Mathematics* (AM) and *Models for Quantifying Risk* (MQR) for Candidates Taking Exam 3," Study Note, 2006. This study note is not required but may be helpful.	Notational Differences	A1-A4, C1, C2	W

Source Key

L May be purchased from the publisher or bookstore or borrowed from the CAS Library.

NEW Indicates new or updated material or modified citation.

W Represents material in the 2010 Web Notes that is available at no charge from the "Study Tools" section of the CAS Web Site. A printed version may be purchased from the CAS Online Store.

Publishers and Distributors for Exam 3L

Contact information is furnished for those who wish to purchase the text references cited for Exam 3L. Publishers and distributors are independent and listed for the convenience of candidates; inclusion does not constitute endorsement by the CAS.

ACTEX Publications, 107 Groppo Drive, Suite A, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; e-mail: retail@actexmadriver.com; Web site: www.actexmadriver.com.

Actuarial Bookstore, P.O. Box 69, Greenland, NH 03840; telephone: (800) 582-9672 (U.S. only) or (603) 430-1252; fax: (603) 430-1258; Web site: www.actuarialbookstore.com.

Bowers, N.L.; Gerber, H.U.; Hickman, J.C.; Jones, D.A.; and Nesbitt, C.J., *Actuarial Mathematics* (Second Edition), 1997, Society of Actuaries, 475 N. Martingale Road, Suite 600, Schaumburg, IL 60173-2226; telephone: (847) 706-3500; fax: (847) 706-3599; Web site: www.soa.org.

Cunningham, R.; Herzog, T.; and London, R, *Models for Quantifying Risk* (Third Edition), 2008, ACTEX Publications, 140 Willow Street, Suite One, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; Web site: www.actexmadriver.com; e-mail: retail@actexmadriver.com.

Hogg, R.V.; Craig, A.T.; and McKean, J.W., *Introduction to Mathematical Statistics* (Sixth Edition), 2004, Prentice Hall, Inc., 200 Old Tappan Road, Old Tappan, NJ 07675; telephone: (800) 282-0693; Web site: www.prenhall.com.

Hogg, R.V.; and Tanis, E., *Probability and Statistical Inference* (Eighth Edition), 2010, Prentice Hall, Inc., 200 Old Tappan Road, Old Tappan, NJ 07675; telephone: (800) 282-0693; Web site: www.prenhall.com.

Larsen, R.J.; and Marx, M.L., *An Introduction to Mathematical Statistics and Its Applications* (Fourth Edition), 2006, Prentice Hall, Inc., 200 Old Tappan Road, Old Tappan, NJ 07675; telephone: (800) 282-0693; Web site: www.prenhall.com.

Mad River Books (A division of ACTEX Publications), 140 Willow Street, Suite One, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; e-mail: retail@actexmadriver.com.

McDonald, R.L., *Derivatives Markets* (Second Edition), 2006, Addison Wesley, imprint of Pearson Education, Inc., 200 Old Tappan Road, Old Tappan, NJ 07675; Web site: http://www.aw-bc.com/catalog/.

SlideRule Books, P.O. Box 69, Greenland, NH 03840; telephone: (877) 407-5433 or (603) 373-6140; fax: (877) 417-5433 or (603) 430-1258; Web site: www.sliderulebooks.com.