Western Canada Linear Algebra Meeting

At Washington State University, Pullman, WA

May 26–27, 2018

Supported by

Pacific Institute for Mathematical Sciences
International Linear Algebra Society
Washington State University

Organisers

Shaun Fallat, University of Regina
Hadi Kharaghani, University of Lethbridge
Steve Kirkland, University of Manitoba
Sarah Plosker, Brandon University
Michael Tsatsomeros, Washington State University
Pauline van den Driessche, University of Victoria

Local organisers

Judi McDonald
Michael Tsatsomeros

Location
Smith Center for Undergraduate Education (aka CUE), **room 419**

1 Program

Saturday, May 26

7:30 - 8:25	Registration, coffee
8:25 - 8:30	Welcome
	Chair: Michael Tsatsomeros
8:30 - 9:20	Vlado Nikiforov, The spectral norm and the spectral radius of hypermatrices
9:20 - 9:45	Colin Garnett, Algebraic conditions that preclude SAPpiness
9:45 - 10:10	Jillian Glasset, Spectrally Arbitrary Zero-Nonzero Patterns over Rings with Unity
10:10 - 10:45	Coffee break
	Chair: Judi McDonald
10:45 - 11:10	Pietro Paparella, On the realizability of the critical points of a realizable list
11:10 - 11:35	Mashael Albaidani, Properties of Nonnegative Matrices That are True for General Matrices
11:35 - 12:00	Xavier Martnez-Rivera, The qpr-sequence
12:00 - 1:30	Group picture and lunch
	Chair: Shaun Fallat
1:30 - 2:20	Jenna Rajchgot, On the algebra generated by three commuting matrices
2:20 - 2:45	Tin-Yau Tam, Matrix Inequalities and Their Extensions to Lie Groups
2:45 - 3:10	Patrick Torres, Spectra of Matrices and Convex Hulls of Matrix Powers
3:10 - 3:45	Coffee break
	Chair: Steve Kirkland
3:45 - 4:10	Joseph D. Fehribach, Kirchhoff Graphs, Construction and Uniformity
4:10 - 4:35	Amy Yielding, An Investigation of the Relationship Between the Inertia Table of a Graph and its Clique Graph
4:35 - 5:00	Enzo Wendler, α-adjacency: A generalization of adjacency matrices

Sunday, May 27

8:00 - 8:30	Coffee
	Chair: Pauline van den Driessche
8:30 - 8:55	Jeff Stuart, Algebra and Eigenvalues
8:55 - 9:20	Parthasarathi Nag, On the existence of projective representation arising from central extension of Witt Algebra using Gelfand-Fuchs 2-cocycle
9:20 - 9:45	Mohsen Aliabadi, On matching property in vector spaces
9:45 - 10:10	Faith Zhang, Rank one perturbation and its applications
10:10 - 10:45	Coffee
	Chair: Michael Tsatsomeros
10:45 - 11:10	Megan Wendler, Semimonotone Matrices
11:10 - 11:35	Paul Zachlin, Household Eigenvalue Inclusion Regions
11:35 - 12:00	Louis Deaett, Matroid theory and matrix minimum rank
12:00	Farewell

2 Abstracts

Mashael Albaidani, WSU

Properties of Nonnegative Matrices That are True for General Matrices

In this talk we will explore properties established for nonnegative matrices that hold in a more general context. In particular, we show that if a matrix A has the property that $\text{index}_0(A) \leq 1$, and q is a positive integer such that for all distinct eigenvalues λ and μ, it follows that $\lambda^q \neq \mu^q$, then there is a permutation matrix P such that $P^{-1}AP$ and $P^{-1}A^qP$ are in Frobenius normal form with the same block partitioning. In more general case, if $A \in M_n(\mathbb{R})$, $\text{index}_0(A) \leq 1$, and q is a positive integer such that for all distinct eigenvalues λ and μ, it follows that $\lambda^q \neq \mu^q$, then for every an invertible matrix S such that $S^{-1}A^qS$ in block upper triangular form then $S^{-1}AS$ is in block upper triangular form with the same partition. The latter result is used in the context of cone theory.

Mohsen Aliabadi, University of Illinois at Chicago

On matching property in vector spaces

A matching in an Abelian group G is a bijection f from a subset A to a subset B in G such that $a + f(a) \notin A$, for all $a \in A$. This notion was introduced by Fan and Losonczy who used matchings in $\mathbb{Z}^n$ as a tool for studying an old problem of Wakeford concerning elimination of monomials in a generic homogenous form under a linear change of variables . We show a sufficient condition for the existence of matchings in arbitrary groups and its linear analogue, which lead to some generalizations of the existing results in the theory of matchings in groups and central extensions of division rings. We introduce the notion of relative matchings between arrays of elements in groups and use this notion to study the behavior of matchable sets under group homomorphisms.

Louis Deaett, Quinnipiac University

Matroid theory and matrix minimum rank

Given only which entries of a matrix are zero and which are nonzero, what is the smallest possible rank the matrix might have? That is the classical minimum rank problem for zero-nonzero matrix patterns. We show how to generalize this problem to the setting of matroids. The original problem, for matrices over a specific field, becomes the restriction of the generalized problem to the class of matroids representable over that field – at least when the field is infinite. We show how to recover some known results about the matrix minimum rank problem by viewing things through this matroid-theoretic lens. We also use known properties of matroid representability to obtain results on how the matrix minimum rank may depend on the field chosen.

Joseph D. Fehribach, Worcester Polytechnic Institute

Kirchhoff Graphs, Construction and Uniformity

Kirchhoff graphs are graphs with vector edges which display the orthocomplementarity of the null space and the row space of a given matrix with rational entries. If this matrix is the stoichiometric matrix for a reaction network or in fact any process, then a Kirchhoff graph for the matrix is a circuit diagram for the network. This talk discusses Kirchhoff graphs, their construction, and their uniformity, which means that with one exception, the number of edge vectors of each type is the same for all types. This is joint work with Judi McDonald, Tyler Reese, Marcel Gietzmann-Sanders, Brigitte Servatius and Randy Paffenroth.

Colin Garnett, Black Hills State University

Algebraic conditions that preclude SAPpiness

This talk focuses on several algebraic conditions on the coefficients of the characteristic polynomial that can be exploited to show that a pattern is not spectrally arbitrary over any field. Using Sage we were able to show that no zero-nonzero pattern with $2n1$ nonzero entries will be spectrally arbitrary over $\mathbb{C}$ where $n \leq 6$. When $n = 7$ we find two zero-nonzero patterns that do not satisfy our algebraic conditions precluding them from being spectrally arbitrary. This talk will discuss the implications of our test and some details of using sage to search for SAPs.

Jillian Glassett, WSU

Spectrally Arbitrary Zero-Nonzero Patterns over Rings with Unity

A zero-nonzero pattern $\mathcal{A}$ is a square matrix with entries $\{0, *\}$. A pattern $\mathcal{A}$ is spectrally arbitrary over $\mathcal{R}$, a commutative ring with unity, if for each n-th degree monic polynomial $f(x) \in \mathcal{R}[x]$, there exists a matrix A over $\mathcal{R}$ with pattern $\mathcal{A}$ such that the characteristic polynomial $p_A(x) = f(x)$. A pattern $\mathcal{A}$ is relaxed spectrally arbitrary over $\mathcal{R}$ if for each n-th degree monic polynomial $f(x) \in \mathcal{R}[x]$, there exists a matrix A over $\mathcal{R}$ with either pattern $\mathcal{A}$ or a subpattern of $\mathcal{A}$ such that the characteristic polynomial $p_A(x) = f(x)$. We evaluated how the structure of rings affects how we determine if a pattern is spectrally arbitrary. We consider whether a pattern $\mathcal{A}$ that is spectrally arbitrary over $\mathcal{R}$ is spectrally arbitrary or relaxed spectrally arbitrary over another commutative ring unity, $\mathcal{S}$. In particular, we discovered that a pattern that is spectrally arbitrary over $\mathbb{Z}$ is relaxed spectrally arbitrary over $\mathbb{Z}/(m)$ for all $m \in \mathbb{Z}_+$ and spectrally arbitrary over $\mathbb{Q}$.

Parthasarathi Nag, Black Hills State University

On the existence of projective representation arising from central extension of Witt Algebra using Gelfand-Fuchs 2-cocycle

We will reconstruct from the conformal group in two dimensions it's algebra of infinitesimal conformal transformations known as the Witt algebra and also from the perspective of Lie algebra of the group of diffeomorphisms of the unit circle. We then obtain the Virasoro algebra via central extension of the Witt algebra using Gelfand-Fuchs 2-cocyle and show how projective representation naturally arises from this construction.

Vladimir Nikiforov, University of Memphis

The spectral norm and the spectral radius of hypermatrices

An r-matrix is a function defined on the Cartesian product of r finite sets. An r-matrix is called symmetric if it is defined on the Cartesian product of r identical sets and is invariant with respect to permutations of the variables.

The linear function L_A of an r-matrix A extends the bilinear form of 2-matrices; likewise, the polynomial form P_A of a symmetric r-matrix extends the quadratic form of symmetric 2-matrices.

Similarly to 2-matrices, the spectral p-norm and the p-spectral radius of r-matrices are defined for every $p \geq 1$ as the maxima of $|L_A|$ and $|P_A|$ over unit tori and spheres in the l_p-norm.

This talk will discuss some results on the spectral p-norm and the p-spectral radius of r-matrices, which extend classical results of matrix theory. Particular attention will be given to nonnegative r-matrices, and to Perron-Frobenius-type results.

Pietro Paparella, University of Washington - Bothel

On the realizability of the critical points of a realizable list

The nonnegative inverse eigenvalue problem (NIEP) is to characterize the spectra of entrywise nonnegative matrices. A finite multiset (herein list) of complex numbers is called realizable if it is the spectrum of an entrywise nonnegative matrix. Monov conjectured that the kth-moments of the list of critical points of a realizable list are nonnegative. Johnson further conjectured that the list of critical points must be realizable. In this talk, Johnsons conjecture, and consequently Monovs conjecture, is established for a variety of important cases including Ciarlet spectra, Suleimanova spectra, spectra realizable via companion matrices, and spectra realizable via similarity by a complex Hadamard matrix. Additionally, we prove a result on differentiators and trace vectors, and use it to provide an alternate proof of a result due to Malamud and a generalization of a result due to Kushel and Tyaglov on circulant matrices. Implications for further research are discussed.

Jenna Rajchgot, University of Saskatchewan

On the algebra generated by three commuting matrices

In the early 1960s, Gerstenhaber proved that the algebra generated by two commuting $d \times d$ matrices has vector space dimension at most d. The analog of this statement for four or more commuting matrices is false. The three matrix version remains open.

After providing some history and context, I'll translate this three commuting matrix statement into an equivalent statement about certain modules over polynomial rings, and prove that this commutative-algebraic reformulation is true in special cases. I'll end with some combinatorial questions about plane partitions, which, if answered, would settle the three matrix analog of Gerstenhaber's theorem for other infinite families of examples.

This is joint work with Matthew Satriano.

Xavier Martinez-Rivera, Auburn University

The qpr-sequence

A *principal* minor of a matrix is the determinant of a (square) submatrix whose row and column indices are the same. The *enhanced principal rank characteristic sequence* (*epr-sequence*) of a symmetric matrix $B \in F^{n\times n}$ is $\ell_1\ell_2\cdots\ell_n$, where ℓ_k is $\mathtt{A}$ (respectively, $\mathtt{N}$) if all of (respectively, none of) the principal minors of order k are nonzero; if some but not all are nonzero, then $\ell_k = \mathtt{S}$. Due to the numerous applications of principal minors, epr-sequences have received considerable attention since their introduction.

An *almost-principal* minor of a matrix is the determinant of a (square) submatrix whose row and column indices differ in exactly one index. Motivated by the fact that principal and almost-principal minors have applications in algebraic geometry, statistics, theoretical physics and matrix theory, for example, we have introduced a new sequence that extends the epr-sequence by also taking into consideration the almost-principal minors of the matrix. A minor of a matrix is *quasi-principal* if it is a principal or an almost-principal minor. The *quasi principal rank characteristic sequence* (*qpr-sequence*) of a symmetric matrix $B \in F^{n\times n}$ is $q_1q_2\cdots q_n$, where q_k is $\mathtt{A}$ (respectively, $\mathtt{N}$) if all of (respectively, none of) the quasi-principal minors of order k are nonzero; if some but not all are nonzero, then $q_k = \mathtt{S}$.

A necessary condition for the attainability of a qpr-sequence is presented in this talk, which concludes by giving particular attention to the qpr-sequences of symmetric matrices over fields of characteristic 0, of which a complete characterization will be presented. This characterization establishes a contrast between qpr- and epr-sequences, as the latter are still far from being characterized.

Jeff Stuart, Pacific Lutheran University

Algebra and Eigenvalues

Everyone who survives a first course in matrix theory knows that if $\mathbf{A}$ is an $n \times n$ matrix over the real or complex numbers, then there is a unique monic polynomial $p(x)$ of degree n such that $p(\mathbf{A})$ is the zero matrix, and consequently, the n roots of $p(x)$ are the eigenvalues of $\mathbf{A}$ accounting for multiple roots. Computing $p(x)$ from $\mathbf{A}$ (let alone extracting its roots), however, is computationally nontrivial. Nonetheless, the search for algebraic relationships that reveal information about the eigenvalues can be very fruitful. For example, if $\mathbf{A}$ satisfies the algebraic relationship $\mathbf{P}_n\mathbf{A} = \mathbf{A}\mathbf{P}_n$ where $\mathbf{P}_n$ is the $n \times n$ irreducible circulant permutation matrix for which $(\mathbf{P}_n)^{n-1} = (\mathbf{P}_n)^{-1} = (\mathbf{P}_n)^T$, then the eigenvalues of $\mathbf{A}$ are immediately determined in a trivial manner from the entries in the first row of $\mathbf{A}$. We examine several generalizations of the circulant relationship, namely, $\mathbf{R}\mathbf{A} = \theta\mathbf{A}^{s+1}\mathbf{R}$ where θ is a unimodular complex number, s is a nonnegative integer, and $\mathbf{R}$ is a square matrix satisfying $\mathbf{R}^k = \mathbf{I}_n$ for some minimum positive integer k. Some interesting results on the spectrum and structure of $\mathbf{A}$ from joint work by Minerva Catral, Leila Lebtahi, Nestor Thome and the presenter will be presented.

Tin-Yau Tam, Auburn University/University of Nevada - Reno

Matrix Inequalities and Their Extensions to Lie Groups

We will discuss some classical matrix inequalities and their extensions that are in my new book "Matrix Inequalities and Their Extensions to Lie Groups.

Patrick K. Torres, WSU

Spectra of Matrices and Convex Hulls of Matrix Powers

Let A be a square matrix of order n. Invertibility of all convex combinations of A and the identity matrix, I, is necessary and sufficient for all real eigenvalues of A to be positive. Analogously, the invertibility of all convex combinations of the rows of A and the corresponding rows of I is necessary and sufficient for A to be a P-matrix, i.e., a matrix whose principal minors are all positive. We present a generalization of these results by considering convex combinations of higher powers of A and of their rows. The invertibility of matrices in these convex hulls is associated with the eigenvalues of A lying in open sectors of the right half-plane and provide a general context for the theory of matrices with P-matrix powers. We present a new result regarding the spectrum of A that holds as a consequence of assuming the invertibility of all infinite convex combinations of A and its powers.

Enzo Wendler, WSU

α-adjacency: A generalization of adjacency matrices

B. Shader and W. So extended the idea of a standard adjacency matrix to the skew adjacency matrix in which an orientation δ is given to a simple undirected graph G and a skew adjacency matrix $S(G^\delta)$ is created. The α adjacency matrix further extends this idea to an arbitrary field $\mathbb{F}$. There are many different orientations that can be given to the graph each with its own α spectrum. We also look at averaging the characteristic polynomials to create an average α characteristic polynomial. In particular we derive a Harary Sachs theorem for the average α characteristic polynomial and determine a few features of a graph which can be determined from the average α characteristic polynomial.

Megan Wendler, WSU

Semimonotone Matrices

A matrix A is called semimonotone if for all nonzero $x \geq 0$ there exists a k such that $x_k > 0$ and $(Ax)_k \geq 0$. Besides some basic results on semimonotone matrices, such as the result that a matrix A is semimonotone if and only if A is completely semipositive, much remains unknown about this class of matrices. In this talk, I will discuss and prove some new results on semimonotone matrices that have so far not appeared in the literature. Some conjectures which are believed to be true but have not yet been proven will also be discussed.

Amy Yielding, Eastern Oregon University

An Investigation of the Relationship Between the Inertia Table of a Graph and its Clique Graph

In this talk, I introduce a special family of graphs coined clique graphs, denoted KG. Utilizing zero-forcing sets and clique covers I establish the minimum rank of such KG in relation to G. For special G it can be shown that the inertia table of KG is the same as G with the northeast lemma applied. Lastly I provide the inertia tables for the clique graphs of cycles, paths, stars, and complete graphs.

Paul Zachlin, Lakeland Community College

Household Eigenvalue Inclusion Regions

We study eigenvalue inclusion regions first defined by Householder and explore applications to large, sparse matrices. We also study pseudospectra and relative pseudospectra, which are similarly defined eigenvalue inclusion regions. Then we explore connections between these sets.

Faith Zhang, WSU

Rank one perturbation and its applicationsRank one perturbation and its applications

We consider the spectral effects and eigenspaces of rank-one perturbations of a matrix A. The perturbations are either general or an eigenvector of A (or $A*$) is involved. We discuss applications in the classical pole assignment and stabilization problems in linear control theory, in nonnegative inverse eigenvalue problems, as well as in Perron-Frobenius theory.