FIXED POINTS AND INTEGRAL EQUATIONS

T. A. BURTON

Abstract In 1930 Juliusz Schauder (1899-1943) offered a very general fixed point theorem which has been the standard in many areas of applied science for 85 years. It has two hypotheses which are a challenge to verify in so many potential applications. In 1955 Helmut Schaefer offered a theorem which modified one of those hypotheses and it is that result on which we focus. The applications often involve integral equations from classical and modern areas such as heat transfer, turbulence, population biology, and every inversion of fractional differential equations of either Caputo or Riemann-Liouville type. Important theorems with difficult hypotheses offer great opportunities for research.

While the presentation can cover a wider area, to be definite, this talk will focus on specialization to scalar integral equations of the form

$$x(t) = p(t) - \int_0^t (t-s)^{q-1} f(s, x(s)) ds, \quad 0 < q < 1, \tag{1}$$

with a goal of reducing the work of **verifying** those two hypotheses to elementary problems within the reach of investigators with two semesters of calculus. The fixed point theorem yields very exact qualitative properties of the solution. The restriction to (1) is still general enough to cover standard problems from the aforementioned areas. For example, fractional problems concern a general $q \in (0, 1)$, while heat problems almost always have $q = 1/2$.

Schauder worked in Lviv (Lwow)(East Poland) (now West Ukraine) and was a colleague of Stefan Banach. Schauder was shot and killed in 1943 when being taken to a death camp. In Lviv the mathematicians would take their coffee break at the Scottish Cafe which kept the Scottish Book of mathematics problems, still available from Amazon.com for $59.89. And the Scottish Cafe still serves coffee to mathematicians.

Schaefer had a brilliant career as a professor in Tubingen, Germany. Universities in the US were upgraded as a result of fear concerning the Russian satellite, Sputnik, and several German professors were invited to WSU on visiting appointments. Schaefer was one of that group and he directed the first Ph.D. dissertation in the WSU mathematics department. He was in East Germany at the end of WWII and received his Ph.D. in mathematics at Leipzig in 1951. Later, he escaped to West Germany, making his appointment to WSU possible.

732 Caroline St., Port Angeles, WA
E-mail address: taburton@olypen.com